Overview of First-Order Logic

Chapter 8

Outline

- Why FOL?
- Syntax of FOL
- Expressing Sentences in FOL
- Wumpus world in FOL
- Knowledge Engineering

Pros and Cons of Propositional Logic (PC)

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- Meaning in PC is *context-independent*
  - Unlike natural language: Compare "Bring me the iron".
    - "iron" could be an instrument for removing creases from clothes, a golf club, a piece of metal, ...
    - "me" depends on who is doing the talking.

Pros and Cons of PC

Cons:

- PC has limited expressive power
    - E.g., cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square

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Functions: E.g. father of, best friend, plus, . . .

Aside: Logics in General

There are lots of logics:

Logic	*Ontological Commitment*	*Epistemological Commitment*
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
		true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known fuzzy value
Modal logic (logic of beliefs)	facts, possible worlds	true/false/unknown + necessarily t/f/unkn
Description logic	concepts, roles, objects	true/false/unknown

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Syntax of FOL: Basic Elements

- Constants: *Wumpus*, 2, *SFU*, ...
- Predicates: *Brother*, *Plus*, ...
- Functions: *Sqrt*, *LeftLegOf*, ...
- Variables: x, y, ...
- Connectives: $\wedge$, $\vee$, $\neg$, $\Rightarrow$, $\equiv$.
- Equality: $=$
- Quantifiers: $\forall$, $\exists$

And, strictly speaking, there is punctuation: "(", ")", ",".

Terms and Atomic Sentences

Basic idea with FOL:

- There are *objects* or *things* in the domain being described.
  - *Terms* in the language denote objects.
  - E.g. $JohnQSmith$, 12, $CMPT310$, $favouriteCatOf(John)$, ...

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- There are *objects* or *things* in the domain being described.
  - *Terms* in the language denote objects.
  - E.g. $JohnQSmith$, 12, $CMPT310$, $favouriteCatOf(John)$, ...
- One makes *assertions* concerning these objects.
  - *Formulas* in the language express assertions.
  - E.g. $Student(JohnQSmith)$,
    $favouriteCatOf(John) = Fluffy$,
    $\forall x.\ BCUniv(x) \Rightarrow (\neg HasMedSchool(x) \vee x = UBC)$

And that's it!

Terms

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- A term can be:
  - a constant, such as *Chris*, car_{54}, ...
  - a function application such as *LeftLegOf*(*Richard*), *Sqrt*(2), *Sqrt*(*Sqrt*(2)), ...
- A term can contain variables
  - When we get to formulas, we'll want variables to be *quantified*
- A term with no variables is called *ground*

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- An atomic sentence is of the form $predicate(term_1, \ldots, term_n)$ or $term_1 = term_2$
- Example atomic sentences (and terms):
  - $Likes(Arvind, ZeNian)$ could be true or false
  - $BrotherOf(Mary, Sue)$ is false (for normal understanding of $BrotherOf$, $Mary$, Sue)
  - $Married(FatherOf(Richard), MotherOf(John))$ could be true or false.
- There may be more than one way to express something. Compare:

  $MotherOf(John, Sue)$ – predicate vs.

  $Sue = MotherOf(John)$ – function.

Complex Sentences

- Complex sentences are made from atomic sentences using the connectives of propositional logic:

  $$\neg S,\ (S_1 \wedge S_2),\ (S_1 \vee S_2),\ (S_1 \Rightarrow S_2),\ (S_1 \equiv S_2)$$

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- Examples:
  - $Red(car_{54}) \wedge \neg Red(car_{54})$
  - $Sibling(Joe, Alice) \Rightarrow Sibling(Alice, Joe)$
  - $King(Richard) \vee King(John)$
  - $King(Richard) \Rightarrow \neg King(John)$
  - $Purchase(p) \wedge$
    $Buyer(p) = John \wedge$
    $ObjectType(p) = Bike$
- Semantics is the same as in propositional logic

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- $Student(John)$ is true or false and says something about a specific individual, John.
- We can be much more flexible if we allow variables which can range over element of the domain.
- Now allow sentences of the form:

$$(\forall x S),\ (\exists x S)$$

  - $(\forall x S)$ is true if no matter what x refers to, S is true.
  - $(\exists x S)$ is true if there is some element of the domain for which S is true.

Universal Quantification

Form: $\forall \langle \mathit{variables} \rangle \langle \mathit{sentence} \rangle$

- Allows us to make statements about all objects that have certain properties.
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- *Roughly* speaking, equivalent to the *conjunction* of *instantiations* of P

$$\begin{array}{ll} (At(Joe, SFU) \Rightarrow Smart(Joe)) & \wedge \\ (At(Alice, SFU) \Rightarrow Smart(Alice)) & \wedge \\ (At(SFU, SFU) \Rightarrow Smart(SFU)) & \wedge \ldots \end{array}$$

- Aside: Formulas are *finite* in length, so universal quantification in general can't be expressed as a big conjunction.

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall x(At(x, SFU) \wedge Smart(x))$$

means

"Everyone is at SFU and everyone is smart"

and not

"Everyone at SFU is smart".

Existential Quantification

Form: $\exists \langle \mathit{variables} \rangle \langle \mathit{sentence} \rangle$

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- *Roughly* speaking, equivalent to the *disjunction* of *instantiations* of P

$$\begin{array}{ll} (At(Joe, UVic) \wedge Smart(Joe)) & \vee \\ (At(Alice, UVic) \wedge Smart(Alice)) & \vee \\ (At(SFU, UVic) \wedge Smart(SFU)) & \vee \ldots \end{array}$$

- But again, we cannot have an infinite disjuntion!

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: Using $\Rightarrow$ as the main connective with $\exists$:

  $$\exists x(At(x, UVic) \Rightarrow Smart(x))$$

  is true if (among other possibilities) there is someone who is not at UVic!
- On the other hand:

  $$\exists x(At(x, UVic) \wedge Smart(x))$$

  is true if there is someone who is at UVic and is smart.

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    "There is a person who loves everyone"
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- *Quantifier duality*: each can be expressed using the other

$$\forall x \; Likes(x, IceCream) \;\equiv\; \neg \exists x \; \neg Likes(x, IceCream)$$
$$\exists x \; Likes(x, Broccoli) \;\equiv\; \neg \forall x \; \neg Likes(x, Broccoli)$$

☞ Like De Morgan's Rule

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  $\forall x, y \ (FirstCousin(x, y) \equiv$
  $\exists p, ps(Parent(p, x) \wedge Sibling(ps, p) \wedge Parent(ps, y)))$

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  $\forall x(Dog(x) \Rightarrow Mammal(x))$.
  $Student(Anne)$.

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- Aside: Better is:

$$\begin{aligned} \forall x, y\ Sibling(x, y) \equiv [\neg(x = y) \land \exists m, f\ (Mother(m, x) \land \\ Father(f, x) \land Mother(m, y) \land Father(f, y))] \end{aligned}$$

$+$ definitions of *Mother* and *Father*.

☞ As with programming, it is important *how* you express a domain.

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- $t_1 = t_2$ says that t_1 and t_2 refer to the same individual
  - $=$ is a relation between *terms*
  - E.g. $CapitalOf(BC) = Victoria$.

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- We also want to *ASK* things of a *KB*,
  $ASK(KB, \exists x\ Student(x))$
  - These are *queries* or *goals*
  - The KB should output x where $Student(x)$ is true:
    $\{x/Alice, \ldots\}$

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- Express by the percept sentence:
  $Tell(KB, Percept([Smell, Breeze, None, None, None], 5))$
- Then:
  $Ask(KB, \exists a Action(a, 5))$
  - I.e., does KB entail any particular actions at $t = 5$?
  - Ask solves this and returns $\{a/Shoot\}$

Knowledge in the Wumpus World

- Need to specify axioms about the wumpus world; for example:
- *"Perception"*

  $\forall b, g, t, m, c \; Percept([Smell, b, g, m, c], t) \Rightarrow Smelt(t)$

  $\forall s, b, t, m, c \; Percept([s, b, Glitter, m, c], t) \Rightarrow AtGold(t)$

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- Q: If we know $Holding(Gold, t)$ can we conclude $Holding(Gold, t+1)$?
  - Ans: No

Representing Information

- Need to remember properties of locations:
  $\forall x, t \; At(Agent, x, t) \wedge Smelt(t) \Rightarrow Smelly(x)$
  $\forall x, t \; At(Agent, x, t) \wedge Breeze(t) \Rightarrow Breezy(x)$
- Need to be careful that *all* information is represented.
  Consider "Squares are breezy near a pit":

Representing Information

- Need to remember properties of locations:
  $\forall x, t\ At(Agent, x, t) \wedge Smelt(t) \Rightarrow Smelly(x)$
  $\forall x, t\ At(Agent, x, t) \wedge Breeze(t) \Rightarrow Breezy(x)$
- Need to be careful that *all* information is represented. Consider "Squares are breezy near a pit":
  - *Diagnostic* rule – infer cause from effect
    $\forall y\ Breezy(y) \Rightarrow \exists x Pit(x) \wedge Adjacent(x, y)$
  - *Causal* rule – infer effect from cause
    $\forall x, y\ Pit(x) \wedge Adjacent(x, y) \Rightarrow Breezy(y)$

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- Need to be careful that *all* information is represented.
  Consider "Squares are breezy near a pit":
  - *Diagnostic* rule – infer cause from effect
    $\forall y\ Breezy(y) \Rightarrow \exists x Pit(x) \wedge Adjacent(x, y)$
  - *Causal* rule – infer effect from cause
    $\forall x, y\ Pit(x) \wedge Adjacent(x, y) \Rightarrow Breezy(y)$
- Neither of these is complete – e.g., the causal rule doesn't say whether squares far away from pits can be breezy
- *Definition* for the *Breezy* predicate:
  $\forall y\ Breezy(y) \equiv [\exists x\ Pit(x) \wedge Adjacent(x, y)]$

Knowledge Engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base.

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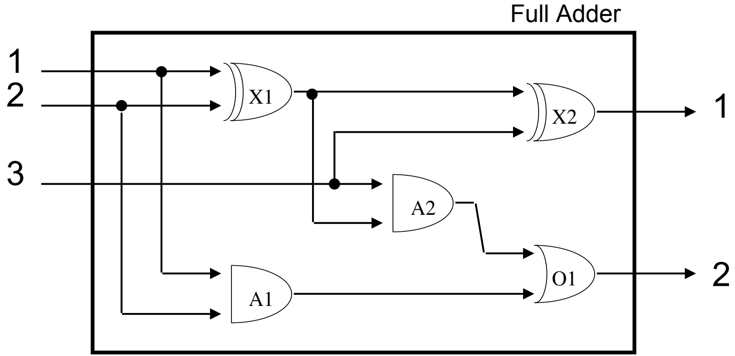
Aside: This is pretty much the same as designing a database schema + instance.

The Electronic Circuits Domain



The Electronic Circuits Domain

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3. Decide on a vocabulary
   - Different possibilities:
     - Function: $Type(X_1) = XOR$
     - Binary predicate: $Type(X_1, XOR)$
     - Unary predicate: $XOR(X_1)$

The Electronic Circuits Domain

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- $\forall p_1, p_2 \ Connected(p_1, p_2) \Rightarrow Connected(p_2, p_1)$
- $\forall g \ \ Type(g) = OR \Rightarrow$
  $Signal(Out(1, g)) = 1 \ \equiv \ \exists n \ Signal(In(n, g)) = 1$

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- $1 \neq 0$
- $\forall p_1, p_2 \; Connected(p_1, p_2) \Rightarrow Connected(p_2, p_1)$
- $\forall g \;\; Type(g) = OR \Rightarrow$
  $Signal(Out(1, g)) = 1 \;\equiv\; \exists n \; Signal(In(n, g)) = 1$
- $\forall g \;\; Type(g) = AND \Rightarrow$
  $Signal(Out(1, g)) = 0 \;\equiv\; \exists n \; Signal(In(n, g)) = 0$

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- $\forall g \ \ Type(g) = OR \Rightarrow$
  $Signal(Out(1, g)) = 1 \ \equiv \ \exists n \ Signal(In(n, g)) = 1$
- $\forall g \ \ Type(g) = AND \Rightarrow$
  $Signal(Out(1, g)) = 0 \ \equiv \ \exists n \ Signal(In(n, g)) = 0$
- $\forall g \ \ Type(g) = XOR \Rightarrow$
  $Signal(Out(1, g)) = 1 \ \equiv \ Signal(In(1, g)) \neq Signal(In(2, g))$

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- $\forall g \;\; Type(g) = OR \Rightarrow$
  $Signal(Out(1, g)) = 1 \;\equiv\; \exists n \; Signal(In(n, g)) = 1$
- $\forall g \;\; Type(g) = AND \Rightarrow$
  $Signal(Out(1, g)) = 0 \;\equiv\; \exists n \; Signal(In(n, g)) = 0$
- $\forall g \;\; Type(g) = XOR \Rightarrow$
  $Signal(Out(1, g)) = 1 \;\equiv\; Signal(In(1, g)) \neq Signal(In(2, g))$
- $\forall g \;\; Type(g) = NOT \Rightarrow Signal(Out(1, g)) \neq Signal(In(1, g))$

The Electronic Circuits Domain

5. Encode the specific problem instance:

$$
\begin{array}{ll}
Type(X_1) = XOR & Type(X_2) = XOR \\
Type(A_1) = AND & Type(A_2) = AND \\
Type(O_1) = OR &
\end{array}
$$

$$
\begin{array}{ll}
Connected(Out(1,X_1), In(1,X_2)) & Connected(In(1,C_1), In(1,X_1)) \\
Connected(Out(1,X_1), In(2,A_2)) & Connected(In(1,C_1), In(1,A_1)) \\
Connected(Out(1,A_2), In(1,O_1)) & Connected(In(2,C_1), In(2,X_1)) \\
Connected(Out(1,A_1), In(2,O_1)) & Connected(In(2,C_1), In(2,A_1)) \\
Connected(Out(1,X_2), Out(1,C_1)) & Connected(In(3,C_1), In(2,X_2)) \\
Connected(Out(1,O_1), Out(2,C_1)) & Connected(In(3,C_1), In(1,A_2))
\end{array}
$$

The Electronic Circuits Domain

6. Pose queries to the inference procedure
   - E.g. what are the outputs, given a set of input signals?
   - I.e.

$$\exists o_1, o_2$$
$$(Signal(In(1, C_1)) = 1 \wedge Signal(In(2, C_1)) = 0 \wedge$$
$$Signal(In(3, C_1)) = 1)$$
$$\Rightarrow$$
$$(Signal(Out(1, C_1)) = o_1 \wedge Signal(Out(2, C_1)) = o_2)$$

The Electronic Circuits Domain

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   - E.g. what are the outputs, given a set of input signals?
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     $\exists o_1, o_2$
     $(Signal(In(1, C_1)) = 1 \wedge Signal(In(2, C_1)) = 0 \wedge$
     $Signal(In(3, C_1)) = 1)$
     $\Rightarrow$
     $(Signal(Out(1, C_1)) = o_1 \wedge Signal(Out(2, C_1)) = o_2)$
7. Debug the knowledge base
   - E.g. may have omitted assertions like $0 \neq 1$.

Summary

- First-order logic:
  - Much more expressive than propositional logic
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- FOL is harder to reason with
  - Undecidable in general