Informed Search Algorithms

Chapter 3.5-6

Outline

Informed Search and Heuristic Functions

- For informed search, we use *problem-specific* knowledge to guide the search.

Topics:

- Best-first search
- A^* search
- Heuristics

Recall: General Tree Search

```
function Tree-Search(problem) returns a solution or failure
   initialize the search tree by the initial state of problem
   loop do {
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion (according to some strategy)
         - remove the leaf node from the frontier
      if the node satisfies the goal state then return the solution
      expand the node and add the resulting nodes to the search tree
   }
```

Informed (Heuristic) Search

- *Idea*: use an *evaluation function* for each node
  - estimate of "desirability" or proximity to a goal.
- Expand the most desirable unexpanded node

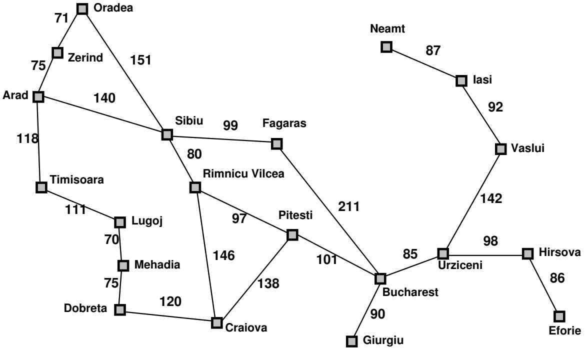
Informed (Heuristic) Search

- *Idea*: use an *evaluation function* for each node
  - estimate of "desirability" or proximity to a goal.
- Expand the most desirable unexpanded node
- Most generally we have:
  - Evaluation function: $f(n) = g(n) + h(n)$
  - $g(n)$ = cost from root to node n
  - $h(n)$ = estimated cost from node n to the goal
    $h(n)$ – *heuristic function*
  - $f(n)$ = estimated total cost of path through n to goal
- Thus for uniform-cost search $f(n) = g(n)$.

Greedy Best-First Search

- Evaluation function $f(n) = h(n)$
  $= \text{estimate of cost from } n \text{ to the closest goal}$
- So, $g(n) = 0$
  - I.e. the cost from the root to n is not considered.
- E.g., $h_{\mathrm{SLD}}(n) = \text{straight-line distance from } n \text{ to Bucharest}$
- Greedy search expands the node that *appears* to be closest to goal

Example: Romania with step costs in km



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
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Fagaras	178
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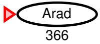
Greedy search example



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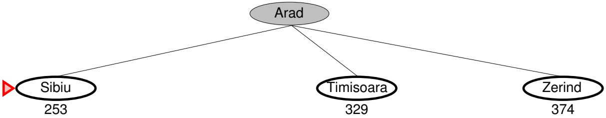

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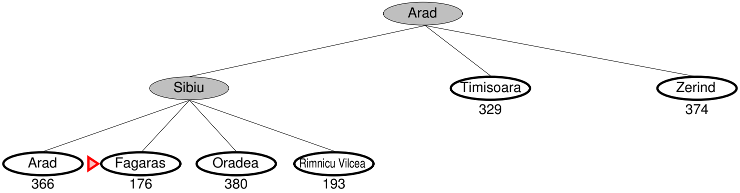

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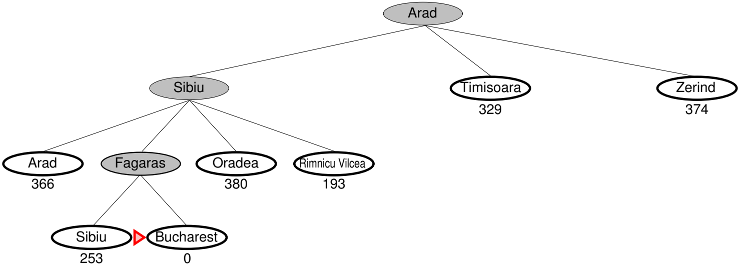

Properties of greedy search

Complete: ??

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Complete: No – can get stuck in loops,

- E.g., with Oradea as goal,
  Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$

☞ Complete in finite space with repeated-state checking

Time: ??

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- Note that this is for an (offline) breadth-first tree-search version of the algorithm.
- An (online) depth-first agent could perform in constant space using via *local* search (later).

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Optimal: No

A* search

Idea:

- Try to avoid expanding paths that look to be expensive
  - Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = cost so far to reach n
  - $h(n)$ = estimated cost to the goal from n
  - $f(n)$ = estimated total cost of path through n to goal
- Expand the node where the cost so far, plus the estimated cost, is minimal.
- Note that $f(n)$ is a heurisitic function. It may not give the best value.
- A good choice of a heurisitic function is crucial for good performance.

A^* search

A^* search (ideally) uses an *admissible* heuristic

- Let $h^*(n)$ be the *true* (unknown) cost from n to the goal.
- A heuristic function $h(n)$ is admissable just if:

$$h(n) \leq h^*(n)$$

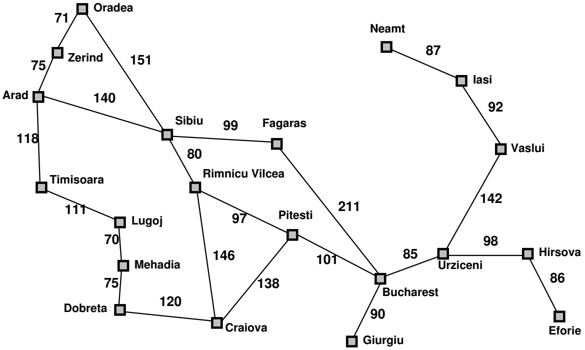
☞ So $h(n)$ never overestimates the cost.

- Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.

E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A^* search is optimal
Corollary: Uniform cost search is optimal (why?)

Example: Romania with step costs in km



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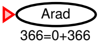

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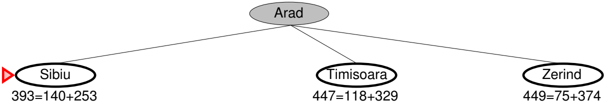

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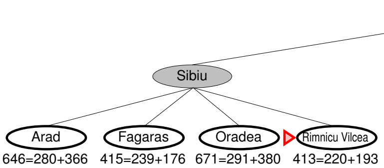

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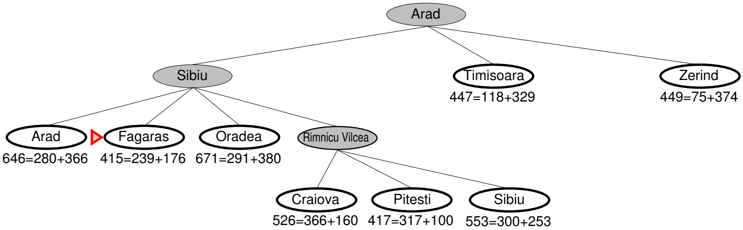

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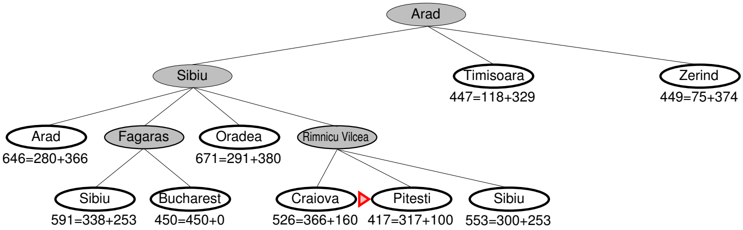

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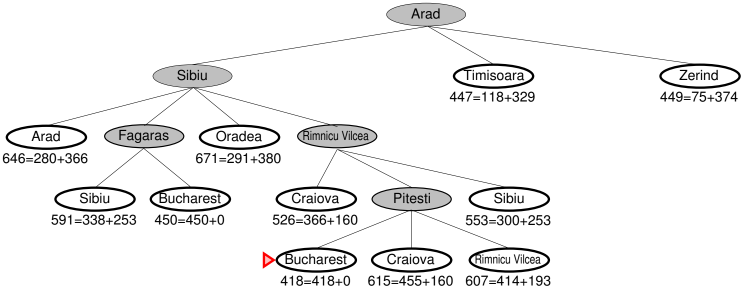
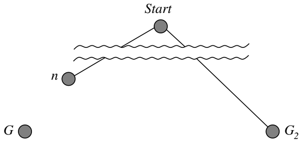


Optimality of A* (standard proof)

- Suppose G_2 is a suboptimal goal.
- Let n be an unexpanded node on a shortest path to an optimal goal G:



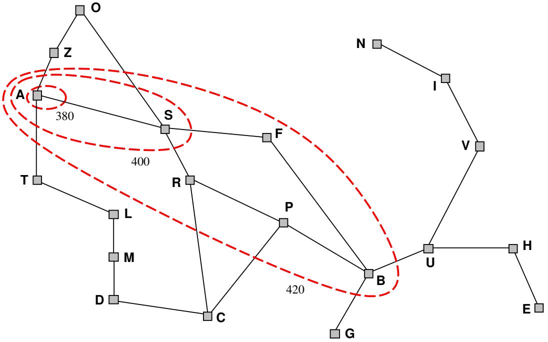
- Then:

$$\begin{array}{rcll} f(G_2) & = & g(G_2) & \text{since } h(G_2) = 0 \\ & > & g(G) & \text{since } G_2 \text{ is suboptimal} \\ & \geq & f(n) & \text{since } h \text{ is admissible} \end{array}$$

- Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (another view)

- *Lemma*: A* expands nodes in order of increasing f value.
- Gradually adds "f-contours" of nodes
  - Cf.: breadth-first adds "layers"
- Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



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Optimal: Yes

- A* expands all nodes with $f(n) < C^*$, where
  $C^* =$ cost of optimal solution
- A* expands some nodes with $f(n) = C^*$
- A* expands no nodes with $f(n) > C^*$

Admissible heuristics

For the 8-puzzle:

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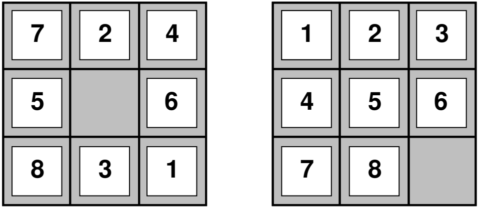

Start State **Goal State**

$h_1(S) = ??$

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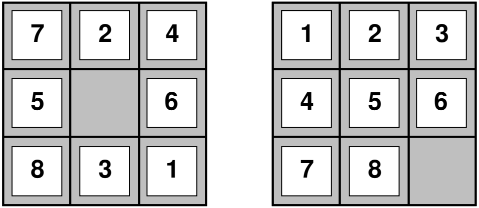

Start State **Goal State**

$h_1(S) = 6$

$h_2(S) = 4+0+3+3+1+0+2+1 = 14$

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 *dominates* h_1, and is better for search
- Typical search costs for 8 puzzle:

$$\begin{array}{ll} d = 14 & \text{IDS} = 3{,}473{,}941 \text{ nodes} \\ & \text{A}^*(h_1) = 539 \text{ nodes} \\ & \text{A}^*(h_2) = 113 \text{ nodes} \\ d = 24 & \text{IDS} \approx 54{,}000{,}000{,}000 \text{ nodes} \\ & \text{A}^*(h_1) = 39{,}135 \text{ nodes} \\ & \text{A}^*(h_2) = 1{,}641 \text{ nodes} \end{array}$$

- For any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Determining admissable heuristic functions

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- E.g.:
  - If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_1(n)$ gives the shortest solution
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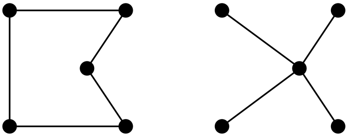
Key point:

The optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Relaxed problems contd.

- Well-known example: *travelling salesperson problem* (TSP)
- Find the shortest tour visiting all cities exactly once



- *Minimum spanning tree* can be computed in $O(n^2)$ and is a lower bound on the shortest (open) tour

Summary: Heuristic functions

- Heuristic functions estimate costs of shortest paths
- Good heuristics can dramatically reduce search cost
- Greedy best-first search expands lowest h
  - incomplete and not always optimal
- A* search expands lowest $g + h$
  - complete and optimal
  - also optimally efficient (up to tie-breaks, for forward search)
- Admissible heuristics can be derived from exact solution of relaxed problems

Local Search: Outline

We consider next *local* search, where we maintain a single current state.

- Iterative improvement algorithms
- Hill-climbing
- Very briefly:
  - Simulated annealing
  - Local beam search

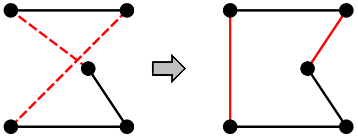
Iterative improvement algorithms

- Idea: In many optimization problems, the *path* to the goal is irrelevant.
  - The goal state itself is the solution
  - E.g. the n-queens problem
- So we may formulate a problem so that:

  state space = set of "complete" configurations
- Examples:
  - find *optimal* configuration, e.g., TSP
  - find configuration satisfying constraints, e.g., timetable
  - also, e.g. propositional satisfiability (SAT)
- In such cases, we can use *iterative improvement* algorithms
  - Keep a single "current" state; try to improve it
  - Uses constant space; suitable for online as well as offline search

Example: Travelling Salesperson Problem

- Start with any complete tour, perform pairwise exchanges



- Variants of this approach get within 1% of optimal very quickly with thousands of cities.

Example: n-queens

- Goal: Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal.

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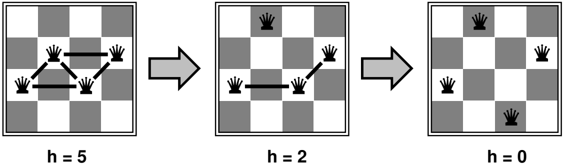
- Goal: Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal.
- Move a queen to reduce number of conflicts.



- Almost always solves n-queens problems almost instantaneously for very large n, e.g., $n = 1,000,000$

Hill-climbing (or gradient ascent/descent)

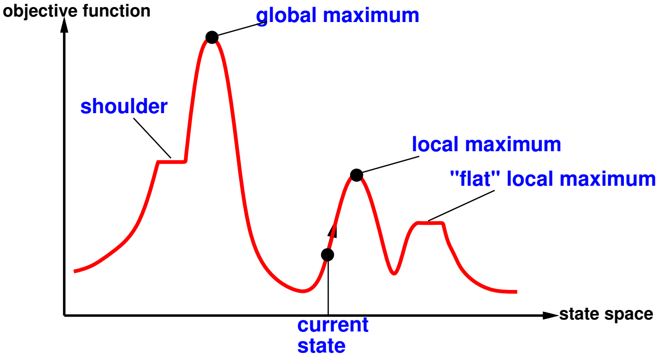
- Idea: Take the best move from a given position
- Aka *greedy local search*.
- "Like climbing a mountain in thick fog with amnesia"

Hill-climbing

```
Function Hill-Climbing(problem) returns a state that is a local
        maximum
  inputs: problem a problem
  local variables: current a node
      neighbor a node
  current ←Make-Node(Initial-State[problem])
  loop do
    neighbor ←a highest-valued successor of current
    if Value[neighbor] ≤ Value[current] then return State[current]
    current ←neighbor
  end
```

Hill-climbing contd.

Useful to consider *state-space landscape*



Hill-climbing contd.

- Hill climbing often gets stuck:

  Local Maxima: I.e. local "peaks".
  E.g. 8-queens gets stuck 86% of the time.

  Ridges: Essentially give a series of local maxima.
  Difficult for hill-climbing to navigate

  Plateaux: A plateau is a flat area in the search space.
  Search degenerates to exhaustive search, or may loop.

Hill-climbing: Strategies if stuck

- *Random-restart hill climbing*: Overcomes local maxima
  - Trivially complete *if* a goal is known to exist.
- *Random sideways moves*: Escape from shoulders but may loop on flat maxima
  - Can also define a hill-climbing version of depth-first search. (But then no longer a *local* search.)

Another Example: Propositional Satisfiabilty

- Goal: Find a *satisfying assignment* for a set of clauses in CNF.
- E.g.

$$(p \vee q \vee \neg r) \wedge (\neg p \vee r) \wedge (\neg p \vee \neg q)$$

is satisfied by setting: $p = \mathit{true}$, $q = \mathit{false}$, $r = \mathit{true}$.

Propositional Satisfiabilty

- Outline of an algorithm:

Function Sat(problem) returns a solution or failure
  Assign truth values arbitrarily to the set of propositional variables
  loop do {
    if the truth assignment satisfies problem
        then return the assignment
    if timeout then return failure
    Find l such that $\bar{l}$ gives the largest increase in clauses satisfied
    Change the truth value of l to $\bar{l}$.
  }

☞ If l is p then $\bar{l}$ is $\neg p$;
if l is $\neg p$ then $\bar{l}$ is p.

Propositional Satisfiabilty

- This algorithm, when proposed in the 1990's, worked *very* well.
- The algorithm also featured random restarts. (I.e. after a while reassign all variable and start over).
  - It handily beat all previous algorithms (notably DPLL).
- Subsequent work in satisfiability has led to huge improvements over the naive greedy algorithm.
- Aside: Another thing that this work pointed out was the importance of choice of test instances.
  - DPLL (and other algorithms) appeared to work well because it turned out they were often tested on easy instances.

Simulated annealing

- Goal: Avoid local maxima
  - Local maxima is the biggest problem with local search.
- Idea: Take a step in a direction other than the best, from time to time.
  - Try to escape local maxima by allowing some “bad” moves *but gradually decrease their size and frequency*
  - These steps are designed to get the solver out of a possible local maximum
- The step size varies.
  - As time passes the step size and probabilty of a non-best step decreases.
- Simulated annealing has proven very effective in a wide range of problems, including VLSI layout, airline scheduling, etc.

Local beam search

Idea:

- Begin with k randomly-generated states.
- Keep k states instead of 1; choose top k of all their successors
- Not the same as k searches run in parallel!
- Searches that find good states recruit other searches to join them

Problem:

Quite often, all k states end up on same local hill

Variant: Stochastic beam search:

Choose k successors randomly, biased towards good ones

- Observe the analogy to natural selection!