Problem Solving and Search

Chapter 3

Outline

- Problem-solving agents
- Problem formulation
- Example problems
- Basic search algorithms

Problem-Solving Agents

In the *simplest* case, an agent will:

- formulate a goal and a problem;
- search for a sequence of actions that solves the problem;
- then execute the actions.

When done it may formulate another goal and start over.

- In this case the performance measure is simply whether or not the goal is attained.

A problem-solving agent uses an *atomic* representation.

Problem-solving agents

Restricted form of general agent:

```
Function Simple-Problem-Solving-Agent(percept) returns an action
    static seq an action sequence, initially empty
        state some description of the current world state
        goal a goal, initially null
        problem a problem formulation
    state ←Update-State(state,percept)
    if seq is empty then
        goal ←Formulate-Goal(state)
        problem ←Formulate-Problem(state,goal)
        seq ←Search(problem)
        if seq = fail then return null
    action ←First(seq,state);    seq ←Rest(seq,state)
    return action
```

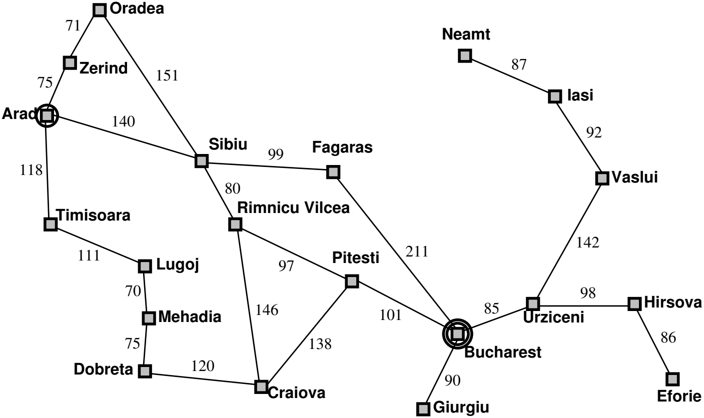
Problem-solving agents

- This is *offline* problem solving, executed "eyes closed."
  - Requires complete knpowledge about the domain
- *Online* problem solving involves acting without necessarily having complete knowledge.

Example: Romania

- On holiday in Romania; currently in Arad.
  - Flight leaves tomorrow from Bucharest
- Formulate *goal*
  - Be in Bucharest
- Formulate *problem*
  - *states*: various cities
  - *actions*: drive between cities
- Find *solution*
  - Sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



Problem Formulation: State-Space Search

A *problem* is defined by five items:

1. The set of *states*, including the *initial state* e.g. "at Arad"
2. *Actions* available to the agent E.g. Vacuum: Suck, Left, ...
3. *Transition model*: What actions do; defines a graph.
   - I.e. $RESULT(s, a) =$ state resulting from doing a in s.
     e.g. $RESULT(In(Arad), Go(Zerind)) = In(Zerind)$

   1.–3. define the *state space*

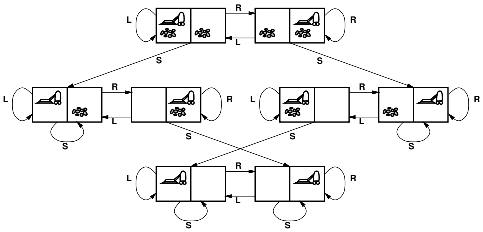
4. *Goal test*. Can be *explicit*, e.g. $x =$ "at Bucharest"
   implicit, e.g. $NoDirt(x)$
5. *Path cost* (additive)
   e.g. sum of distances, number of actions , etc.
   $c(x, a, y)$ is the *step cost*, assumed to be ≥ 0

A *solution* is a sequence of actions from initial state to a goal state.

Selecting a State Space

- The real world is highly complex and contains lots of irrelevant information.
  $\Rightarrow$ state space must be *abstracted* for problem solving
- (Abstract) state will have irrelevant detail removed.
- Similarly, actions must be at the right level of astraction
  - e.g., "Go(Zerind)" omits things like starting the car, steering, etc.
- (Abstract) solution =
  set of real paths that are solutions in the real world

Example: Vacuum World State Space Graph



states:

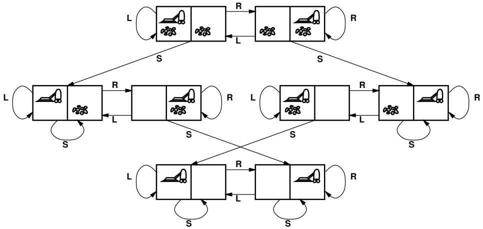
actions:

transition model:

goal test:

path cost:

Example: Vacuum World State Space Graph



states: dirt and robot locations (so 2×2^2 possible states)

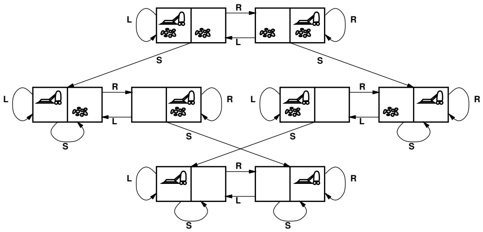
actions:

transition model:

goal test:

path cost:

Example: Vacuum World State Space Graph



states: dirt and robot locations

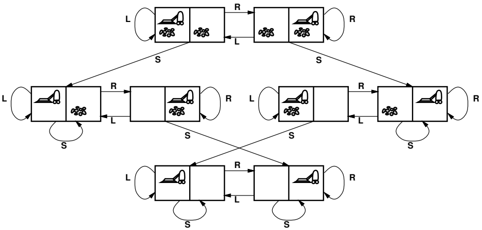
actions: *Left*, *Right*, *Suck*, *NoOp*

transition model:

goal test:

path cost:

Example: Vacuum World State Space Graph



states: dirt and robot locations

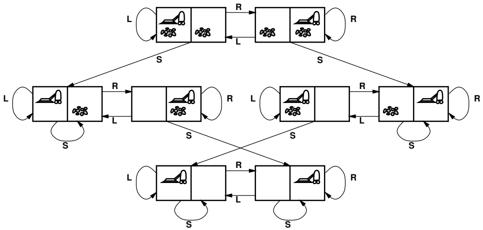
actions: *Left*, *Right*, *Suck*, *NoOp*

transition model: actions as expected, except moving left (right) in the right (left) square is a *NoOp*

goal test:

path cost:

Example: Vacuum World State Space Graph



states: dirt and robot locations

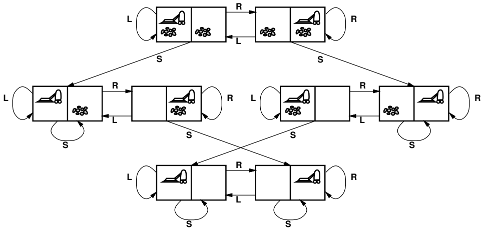
actions: *Left*, *Right*, *Suck*, *NoOp*

transition model: actions as expected, except moving left (right) in the right (left) square is a *NoOp*

goal test: no dirt

path cost:

Example: Vacuum World State Space Graph



states: dirt and robot locations

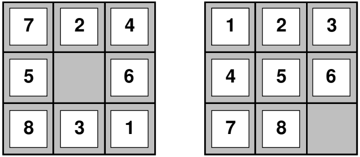
actions: *Left*, *Right*, *Suck*, *NoOp*

transition model: actions as expected, except moving left (right) in the right (left) square is a *NoOp*

goal test: no dirt

path cost: 1 per action (0 for *NoOp*)

Example: The 8-puzzle



Start State   Goal State

states:

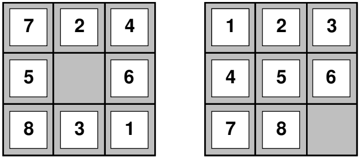
actions:

transition model:

goal test:

path cost:

Example: The 8-puzzle



Start State   Goal State

states: (integer) locations of tiles.

☞ Ignore intermediate positions

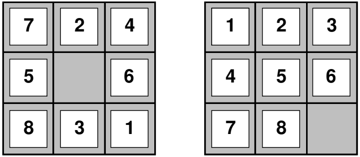
actions:

transition model:

goal test:

path cost:

Example: The 8-puzzle



Start State **Goal State**

states: locations of tiles

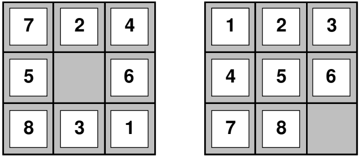
actions: move blank left, right, up, down

transition model:

goal test:

path cost:

Example: The 8-puzzle



Start State **Goal State**

states: locations of tiles

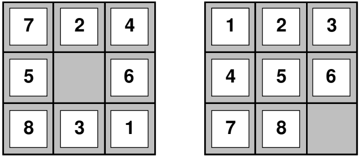
actions: move blank left, right, up, down

transition model: given a state and action give the resulting state

goal test:

path cost:

Example: The 8-puzzle



Start State **Goal State**

states: locations of tiles

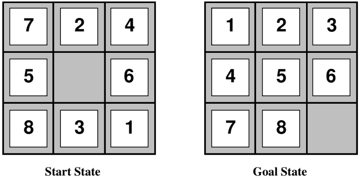
actions: move blank left, right, up, down

transition model: given a state and action give the resulting state

goal test: $=$ goal state (given)

path cost:

Example: The 8-puzzle



states: locations of tiles

actions: move blank left, right, up, down

transition model: given a state and action give the resulting state

goal test: = goal state (given)

path cost: 1 per move

[Aside: optimal solution of *n*-Puzzle family is NP-hard]

Example: Airline Travel

states: Include locations (airports), current time.

- Also perhaps fares, domestic/international, and other "historical aspects".

initial state: Given by a user's query

actions: Flight from current location with attributes such as seat class, departure time, etc.

transition model: The state resulting from taking a flight, including destination and arrival time.

goal test: At the final destination?

path cost: Depends on total cost, time, waiting time, seat type, type of plane, etc.

Others Examples

How about:

- Crosswords?
- n-Queens?
- Propositional Satisfiability?
- Coffee and Mail Delivering Robot?
- Others?

Tree Search Algorithms

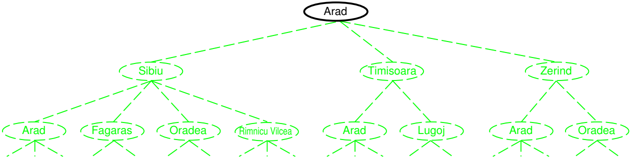
Basic idea:

- Offline exploration of the state space
- Generate successors of already-explored states (a.k.a. *expanding* states)

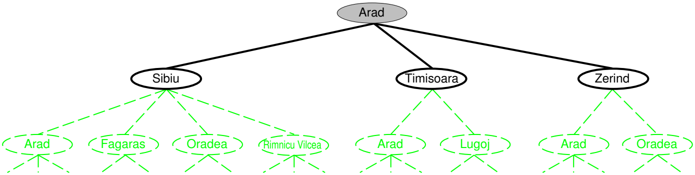
$\Rightarrow$ The set of nodes available for expansion is the *fringe* or *frontier*.

- Key issue: Which node should be expanded next?

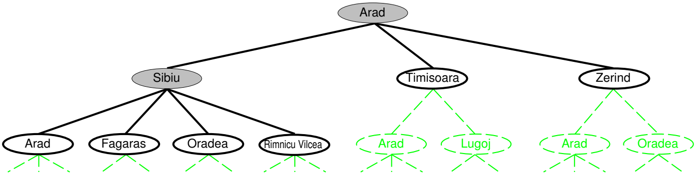
Tree search example



Tree search example



Tree search example



Implementation: General Tree Search

In outline:

```
Function Tree-Search(problem) returns a solution or failure
  Initialize the search tree by the initial state of problem
  loop do {
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion (according to some strategy)
      - remove the leaf node from the frontier
    if the node satisfies the goal state then return the solution
    expand the node and add the resulting nodes to the search tree
  }
```

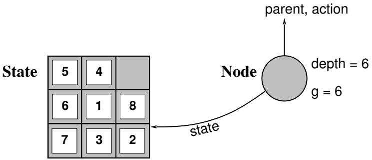
Aside: *Strategy* will most often be implicit in the resulting function.

Implementation: States vs. Nodes

It is important to distinguish the *state space* and the *search tree*.

- A *state* represents a configuration in the problem space.
- A *node* is part of a search tree.
  - has attributes *parent*, *children*, *depth*, *path cost* $g(x)$.

States do not have parents, children, depth, or path cost (though one state may be reachable from another).



An EXPAND function creates new nodes, filling in the various fields and using a SUCCESSORFN of the problem to create the corresponding states.

Search strategies

- A *strategy* is defined by picking the *order of node expansion*
- The *fringe* (also *frontier*) is a list of nodes that have been generated but not yet expanded.

Search strategies

- A *strategy* is defined by picking the *order of node expansion*
- The *fringe* (also *frontier*) is a list of nodes that have been generated but not yet expanded.
- Strategies are evaluated along the following dimensions:
  completeness – does it always find a solution if one exists?

Search strategies

- A *strategy* is defined by picking the *order of node expansion*
- The *fringe* (also *frontier*) is a list of nodes that have been generated but not yet expanded.
- Strategies are evaluated along the following dimensions:

  completeness – does it always find a solution if one exists?

  time complexity – number of nodes generated/expanded

Search strategies

- A *strategy* is defined by picking the *order of node expansion*
- The *fringe* (also *frontier*) is a list of nodes that have been generated but not yet expanded.
- Strategies are evaluated along the following dimensions:
  completeness – does it always find a solution if one exists?
  time complexity – number of nodes generated/expanded
  space complexity – maximum number of nodes in memory

Search strategies

- A *strategy* is defined by picking the *order of node expansion*
- The *fringe* (also *frontier*) is a list of nodes that have been generated but not yet expanded.
- Strategies are evaluated along the following dimensions:

  completeness – does it always find a solution if one exists?
  time complexity – number of nodes generated/expanded
  space complexity – maximum number of nodes in memory
  optimality – does it always find a least-cost solution?

Search strategies

- A *strategy* is defined by picking the *order of node expansion*
- The *fringe* (also *frontier*) is a list of nodes that have been generated but not yet expanded.
- Strategies are evaluated along the following dimensions:
  - *completeness* – does it always find a solution if one exists?
  - *time complexity* – number of nodes generated/expanded
  - *space complexity* – maximum number of nodes in memory
  - *optimality* – does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - b – maximum *branching factor* of the search tree
  - d – depth of the least-cost solution
  - m – maximum depth of the state space (may be ∞)

Uninformed search strategies

- *Uninformed* strategies use only the information available in the problem definition
- I.e. except for the goal state, there is no notion of one state being "better" than another.
- Examples:

Uninformed search strategies

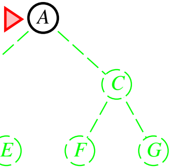
- *Uninformed* strategies use only the information available in the problem definition
- I.e. except for the goal state, there is no notion of one state being "better" than another.
- Examples:
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search

Breadth-first search

Expand the shallowest unexpanded node

Implementation

fringe is a FIFO queue, i.e., new successors go at end

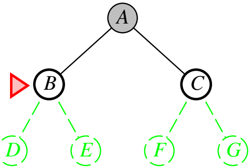


Breadth-first search

Expand the shallowest unexpanded node

Implementation

fringe is a FIFO queue, i.e., new successors go at end

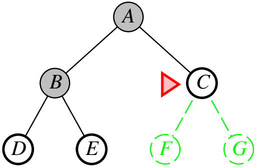


Breadth-first search

Expand the shallowest unexpanded node

Implementation

fringe is a FIFO queue, i.e., new successors go at end

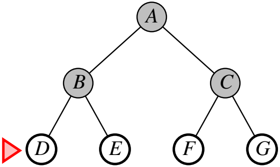


Breadth-first search

Expand the shallowest unexpanded node

Implementation

fringe is a FIFO queue, i.e., new successors go at end



Properties of breadth-first search

Complete: ??

Properties of breadth-first search

Complete: Yes (if b is finite)

Time: ??

Properties of breadth-first search

Complete: Yes (if b is finite)

Time: $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$
I.e., exponential in d

Space: ??

Properties of breadth-first search

Complete: Yes (if b is finite)

Time: $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$
I.e., exp. in d

Space: $O(b^d)$ (keeps every node in memory)

Optimal: ??

Properties of breadth-first search

Complete: Yes (if b is finite)

Time: $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$
I.e., exp. in d

Space: $O(b^d)$ (keeps every node in memory)

Optimal: Yes (if cost $=$ 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at 100MB/sec.
So 24hrs $=$ 8640GB.

Uniform-Cost Search

- Expand the least-cost unexpanded node
- *Implementation*
  fringe = queue ordered by path cost, lowest first
- Equivalent to breadth-first if step costs all equal
- For the travel-in-Romania example, expand the node on the fringe for that city closest in distance to the city at the root (Arad).

Uniform-Cost Search

Complete: Yes, if step cost $\geq \epsilon$, for ϵ some small positive constant.

- So *NoOps* of cost 0 can be a problem.

Time: $O(b^{\lceil C^*/\epsilon \rceil})$, where C^* is the cost of the optimal solution

Space: $O(b^{\lceil C^*/\epsilon \rceil})$

- Time and space complexity can be worse than b^d.

Optimal: Yes

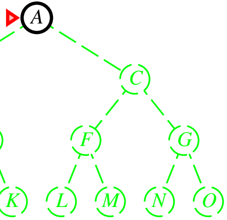
- Nodes expanded in increasing order of $g(n)$ where $g(n)$ is the cost to get to node n.

Depth-First Search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

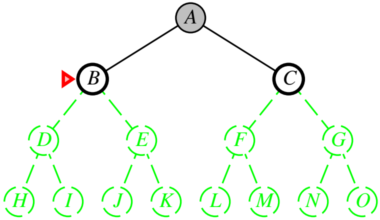


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

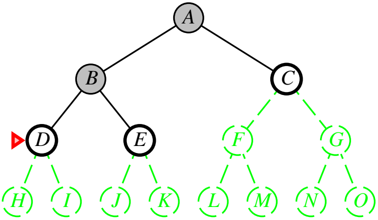


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

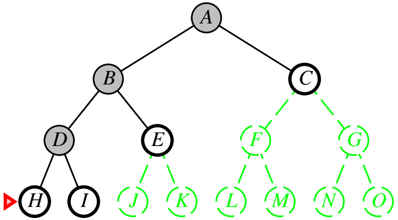


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

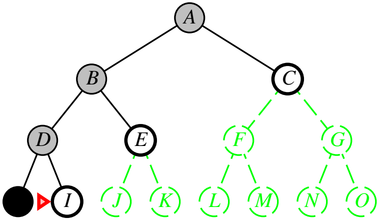


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

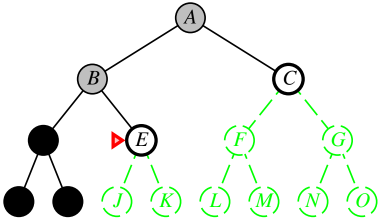


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

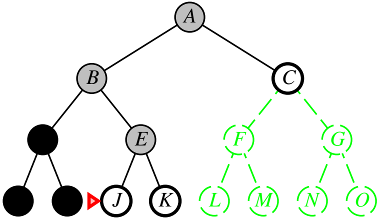


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

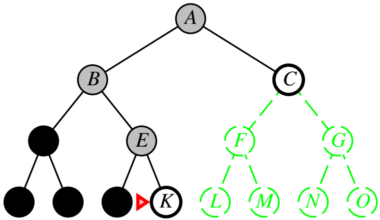


Depth-first search

Expand the deepest unexpanded node

Implementation

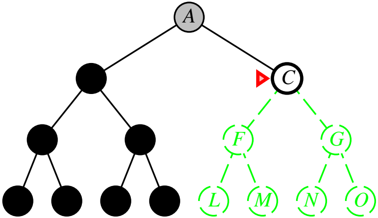
fringe = LIFO queue, i.e., put successors at front



Depth-first search

Expand the deepest unexpanded node

Implementation
    fringe = LIFO queue, i.e., put successors at front

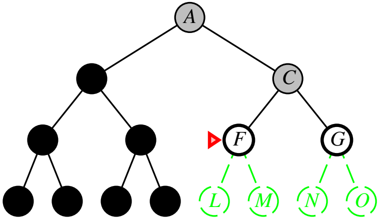


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

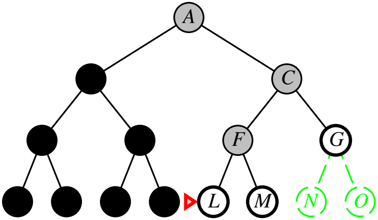


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front

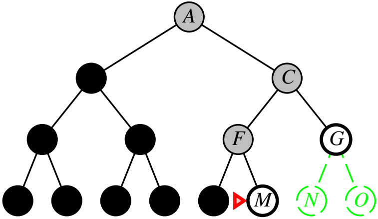


Depth-first search

Expand the deepest unexpanded node

Implementation

fringe = LIFO queue, i.e., put successors at front



Properties of depth-first search

Complete: ??

Properties of depth-first search

Complete: No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time: ??

Properties of depth-first search

Complete: No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time: $O(b^m)$: terrible if m is much larger than d

- But if solutions are dense, may be much faster than breadth-first

Space: ??

Properties of depth-first search

Complete: No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time: $O(b^m)$: terrible if m is much larger than d

- But if solutions are dense, may be much faster than breadth-first

Space: $O(bm)$, i.e., linear space!

Optimal: ??

Properties of depth-first search

Complete: No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time: $O(b^m)$: terrible if m is much larger than d

- But if solutions are dense, may be much faster than breadth-first

Space: $O(bm)$, i.e., linear space!

Optimal: No

Depth-Limited Search

Depth-limited search = depth-first search with depth limit l,

- i.e., nodes at depth l have no successors

Recursive implementation:

The implementation simply calls a "helper" function (described on the next slide):

Function Depth-Limited-Search(problem,limit)
returns soln/fail/cutoff
Recursive-DLS(Make-Node(Initial-State[problem]),
problem,limit)

Depth-Limited Search

Recursive implementation:

```
Function Recursive-DLS(node,problem,limit) returns soln/fail/cutoff
    cutoff-occurred? ←false
    if Goal-Test(problem,State[node]) then return node
    else if Depth[node] = limit then return cutoff
    else for each successor in Expand(node,problem) do
        result ←Recursive-DLS(successor,problem,limit-1)
        if result = cutoff then cutoff-occurred? ←true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

- Note: second edition has a bug in the recursive call!

Iterative Deepening Search

Function Iterative-Deepening-Search(problem) returns a solution
  inputs: problem a problem
  for depth $\leftarrow$ 0 to ∞ do
    result $\leftarrow$Depth-Limited-Search(problem,depth)
    if result $\neq$ cutoff then return result
  end

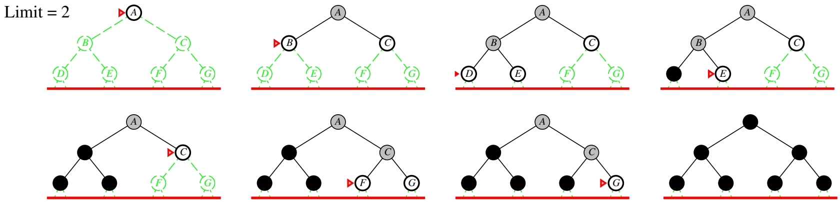
Iterative deepening search $l = 0$

Limit = 0

Iterative deepening search $l = 1$

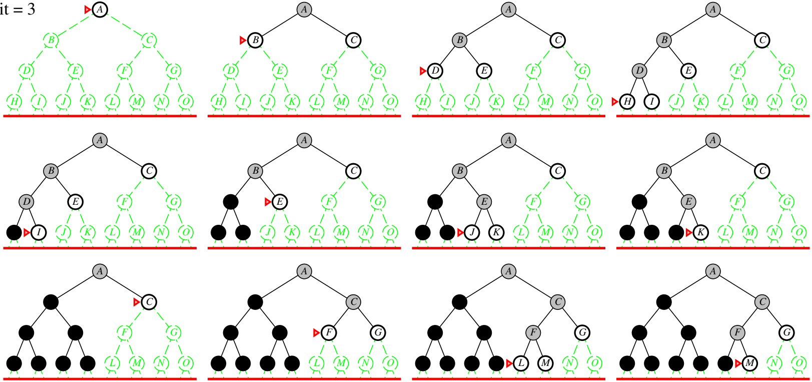
Limit = 1

Iterative deepening search $l = 2$



Iterative deepening search $l = 3$

Limit = 3



Properties of iterative deepening search

Complete: ??

Properties of iterative deepening search

Complete: Yes

Time: ??

Properties of iterative deepening search

Complete: Yes

Time: $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space: ??

Properties of iterative deepening search

Complete: Yes

Time: $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space: $O(bd)$

Optimal:

Properties of iterative deepening search

Complete: Yes

Time: $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space: $O(bd)$

Optimal: Yes, if step cost $= 1$

Properties of iterative deepening search

- Comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$\begin{aligned} N(\text{IDS}) &= 50+400+3,000+20,000+100,000 = 123,450 \\ N(\text{BFS}) &= 10+100+1,000+10,000+100,000 \\ &\quad +999,990 = 111,100 \end{aligned}$$

Properties of iterative deepening search

- Comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$\begin{aligned} N(\text{IDS}) &= 50+400+3,000+20,000+100,000 = 123,450 \\ N(\text{BFS}) &= 10+100+1,000+10,000+100,000 \\ &\quad +999,990 = 111,100 \end{aligned}$$

- For a large search space with unknown depth of solution, IDS is usually best.

Properties of iterative deepening search

- Comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$\begin{aligned} N(\text{IDS}) &= 50+400+3,000+20,000+100,000 = 123,450 \\ N(\text{BFS}) &= 10+100+1,000+10,000+100,000 \\ &\quad +999,990 = 111,100 \end{aligned}$$

- For a large search space with unknown depth of solution, IDS is usually best.
- For BFS, we have the following ratio of IDS to BFS:

b	Ratio
2	3
3	2
5	1.5
10	1.2

Properties of iterative deepening search

- Comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$\begin{aligned} N(\text{IDS}) &= 50+400+3,000+20,000+100,000 = 123,450 \\ N(\text{BFS}) &= 10+100+1,000+10,000+100,000 \\ &\quad +999,990 = 111,100 \end{aligned}$$

- For a large search space with unknown depth of solution, IDS is usually best.
- For BFS, we have the following ratio of IDS to BFS:

b	Ratio
2	3
3	2
5	1.5
10	1.2

- Can be modified to explore uniform-cost tree

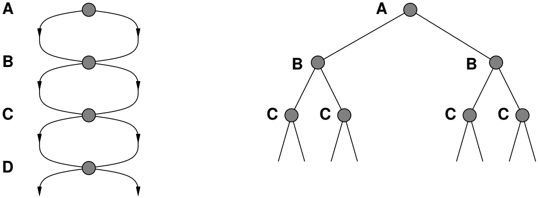
Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

*: If b is finite.

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!



- If we detect repeated states, then our search algorithm amounts to searching a graph rather than a tree.
  - Keep a list of encountered nodes, called the *closed* list.

Graph search

```
Function Graph-Search(problem,fringe) returns a solution, or failure
    closed ←an empty set
    fringe ←Insert(Make-Node(Initial-State[problem]),fringe)
    loop do
        if fringe is empty then return failure
        node ←Remove-Front(fringe)
        if Goal-Test(problem,State[node]) then return node
        if State[node] is not in closed then
            add State[node] to closed
            fringe ←InsertAll(Expand(node,problem),fringe)
    end
```

Summary

- Problem formulation usually requires abstracting from real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms
- Graph search can be exponentially more efficient than tree search