- 8.23 (Raissa D'Souza) For each of the following sentences in English, decide if the FOL sentence is a good translation. If not, explain why not and correct it. (Some sentences may have more than one error!)

  - No two people have the same Social Security Number.

    - $\neg\exists x,y,n\ Person(x) \land Person(y) \Rightarrow [HasSSN(x,n) \land HasSSN(y,n)]$

  - John's SSN is the same as Mary's.

    - $\exists n\ HasSSN(John, n) \land HasSSN(Mary,n)$

  - Everyone's SSN has nine digits.

    - $\forall x,n\ Person(x) \Rightarrow [HasSSN(x,n) \land Digits(n,9)]$

  - Rewrite each sentence above using a function *SSN(x)* instead of the predicate *HasSSN(x,n)*

8.23

a. "No two people have the same social security number."

$$\neg\exists\, x, y, n \;\; Person(x) \land Person(y) \;\Rightarrow\; [HasSS\#(x,n) \land HasSS\#(y,n)].$$

This uses $\Rightarrow$ with $\exists$. It also says that no person has a social security number because it doesn't restrict itself to the cases where x and y are not equal. Correct version:

$$\neg\exists\, x, y, n \;\; Person(x) \land Person(y) \land \neg(x = y) \land [HasSS\#(x,n) \land HasSS\#(y,n)]$$

b. "John's social security number is the same as Mary's."

$$\exists\, n \;\; HasSS\#(John,n) \land HasSS\#(Mary,n).$$

This is OK.

c. "Everyone's social security number has nine digits."

$$\forall\, x, n \;\; Person(x) \;\Rightarrow\; [HasSS\#(x,n) \land Digits(n,9)].$$

This says that everyone has every number. $HasSS\#(x,n)$ should be in the premise:

$$\forall\, x, n \;\; Person(x) \land HasSS\#(x,n) \;\Rightarrow\; Digits(n,9)$$

d. Here $SS\#(x)$ denotes the social security number of x. Using a function enforces the rule that everyone has just one.

$$\neg\exists\, x, y \;\; Person(x) \land Person(y) \;\Rightarrow\; [SS\#(x) = SS\#(y)]$$
$$SS\#(John) = SS\#(Mary)$$
$$\forall\, x \;\; Person(x) \;\Rightarrow\; Digits(SS\#(x), 9)$$

- 8.24 (Yiran Shu) Represent the following sentences in FOL, using a consistent vocabulary (which you must define).
  - Some students took French in Spring 2001.
  - Every student who takes French passes it.
  - Only one student took Greek in Spring 2001.
  - The best score in Greek is always higher than the best score in French.
  - Every person who buys a policy is smart.
  - No person buys an expensive policy.
  - There is an agent who sells policies only to people who are smart.

8.24 In this exercise, it is best not to worry about details of tense and larger concerns with consistent ontologies and so on. The main point is to make sure students understand connectives and quantifiers and the use of predicates, functions, constants, and equality. Let the basic vocabulary be as follows:
$Takes(x, c, s)$: student x takes course c in semester s;
$Passes(x, c, s)$: student x passes course c in semester s;
$Score(x, c, s)$: the score obtained by student x in course c in semester s;
$x > y$: x is greater than y;
F and G: specific French and Greek courses (one could also interpret these sentences as referring to *any* such course, in which case one could use a predicate $Subject(c, f)$ meaning that the subject of course c is field f;
$Buys(x, y, z)$: x buys y from z (using a binary predicate with unspecified seller is OK but less felicitous);
$Sells(x, y, z)$: x sells y to z;
$Shaves(x, y)$: person x shaves person y
$Born(x, c)$: person x is born in country c;
$Parent(x, y)$: x is a parent of y;
$Citizen(x, c, r)$: x is a citizen of country c for reason r;
$Resident(x, c)$: x is a resident of country c;
$Fools(x, y, t)$: person x fools person y at time t;
$Student(x)$, $Person(x)$, $Man(x)$, $Barber(x)$, $Expensive(x)$, $Agent(x)$, $Insured(x)$, $Smart(x)$, $Politician(x)$: predicates satisfied by members of the corresponding categories.

a. Some students took French in spring 2001.
$\exists x\ Student(x) \land Takes(x, F, Spring2001)$.

b. Every student who takes French passes it.
$\forall x, s\ Student(x) \land Takes(x, F, s) \Rightarrow Passes(x, F, s)$.

c. Only one student took Greek in spring 2001.
$\exists x\ Student(x) \land Takes(x, G, Spring2001) \land \forall y\ y \neq x \Rightarrow \neg Takes(y, G, Spring2001)$.

d. The best score in Greek is always higher than the best score in French.
$\forall s\ \exists x\ \forall y\ Score(x, G, s) > Score(y, F, s)$.

e. Every person who buys a policy is smart.
$\forall x\ Person(x) \land (\exists y, z\ Policy(y) \land Buys(x, y, z)) \Rightarrow Smart(x)$.

f. No person buys an expensive policy.
$\forall x, y, z\ Person(x) \land Policy(y) \land Expensive(y) \Rightarrow \neg Buys(x, y, z)$.

g. There is an agent who sells policies only to people who are not insured.
$\exists x\ Agent(x) \land \forall y, z\ Policy(y) \land Sells(x, y, z) \Rightarrow (Person(z) \land \neg Insured(z))$.

h. There is a barber who shaves all men in town who do not shave themselves.
$\exists x\ Barber(x) \land \forall y\ Man(y) \land \neg Shaves(y, y) \Rightarrow Shaves(x, y)$.

i. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.
$\forall x\ Person(x) \land Born(x, UK) \land (\forall y\ Parent(y, x) \Rightarrow ((\exists r\ Citizen(y, UK, r)) \lor Resident(y, UK))) \Rightarrow Citizen(x, UK, Birth)$.

j. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

$$\forall x\ Person(x) \land \neg Born(x, UK) \land (\exists y\ Parent(y, x) \land Citizen(y, UK, Birth)) \Rightarrow Citizen(x, UK, Descent).$$

k. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

$$\begin{aligned}\forall x\ Politician(x) \Rightarrow\ & \\ & (\exists y\ \forall t\ Person(y) \land Fools(x, y, t)) \land \\ & (\exists t\ \forall y\ Person(y) \Rightarrow Fools(x, y, t)) \land \\ & \neg(\forall t\ \forall y\ Person(y) \Rightarrow Fools(x, y, t))\end{aligned}$$

l. All Greeks speak the same language.

$$\forall x, y, l\ Person(x) \land [\exists r\ Citizen(x, Greece, r)] \land Person(y) \land [\exists r\ Citizen(y, Greece, r)] \land Speaks(x, l) \Rightarrow Speaks(y, l)$$

- 9.4 (Adam Bolding-Jones) For each pair of sentences, find a unifying mapping.
  - *P(A,B,B) ; P(x,y,z)*
  - *Q(y, G(A,B)) ; Q(G(x,x), y)*
  - *Older(Father(y),y) ; Older(Father(x), John)*
  - *Knows(Father(y),y) ; Knows(x,x)*

a. $\{x/A, y/B, z/B\}$ (or some permutation of this).

b. No unifier (x cannot bind to both A and B).

c. $\{y/John, x/John\}$.

d. No unifier (because the occurs-check prevents unification of y with $Father(y)$).

- 9.6 (Darren Sturdy) Write down representations for the following sentences in Horn form.
  - Horses, cows and pigs are mammals.
  - An offspring of a horse is a horse.
  - Bluebeard is a horse.
  - Bluebeard is Charlie's parent.
  - Offspring and parent are inverse relations.
  - Every mammal has a parent.

9.6 We use a very simple ontology to make the examples easier:

a. $Horse(x) \Rightarrow Mammal(x)$
$Cow(x) \Rightarrow Mammal(x)$
$Pig(x) \Rightarrow Mammal(x)$.

b. $Offspring(x, y) \wedge Horse(y) \Rightarrow Horse(x)$.

c. $Horse(Bluebeard)$.

d. $Parent(Bluebeard, Charlie)$.

e. $Offspring(x, y) \Rightarrow Parent(y, x)$
$Parent(x, y) \Rightarrow Offspring(y, x)$.
(Note we couldn't do $Offspring(x, y) \Leftrightarrow Parent(y, x)$ because that is not in the form expected by Generalized Modus Ponens.)

f. $Mammal(x) \Rightarrow Parent(G(x), x)$ (here G is a Skolem function).