- (Manshant Singh Kohli) 8.3 and 8.4
  - Is the sentence $\exists x, y: x = y$ valid? Explain.
  - Write down a sentence such that every world in which it is true contains exactly one object.

8.3 The sentence $\exists x, y \;\; x = y$ is valid. A sentence is valid if it is true in every model. An existentially quantified sentence is true in a model if it holds under any extended interpretation in which its variables are assigned to domain elements. According to the standard semantics of FOL as given in the chapter, every model contains at least one domain element, hence, for any model, there is an extended interpretation in which x and y are assigned to the first domain element. In such an interpretation, $x = y$ is true.

8.4 $\forall x, y \;\; x = y$ stipulates that there is exactly one object. If there are two objects, then there is an extended interpretation in which x and y are assigned to different objects, so the sentence would be false. Some students may also notice that any unsatisfiable sentence also meets the criterion, since there are no worlds in which the sentence is true.

- (Xiangpeng Hao) 8.6 Which of the following are valid sentences (i.e. tautologies)?

    - $\exists x\ (x = x) \Rightarrow (\forall y\ \exists z\ y = z)$

    - $\forall x\ P(x) \vee \neg P(x)$

    - $\forall x\ Smart(x) \vee (x = x)$

a. $(\exists x\ x = x) \Rightarrow (\forall y\ \exists z\ y = z)$.

Valid. The LHS is valid by itself—in standard FOL, every model has at least one object; hence, the whole sentence is valid iff the RHS is valid. (Otherwise, we can find a model where the LHS is true and the RHS is false.) The RHS is valid because for every value of y in any given model, there is a z—namely, the value of y itself—that is identical to y.

b. $\forall x\ \ P(x) \lor \neg P(x)$.

Valid. For any relation denoted by P, every object x is either in the relation or not in it.

c. $\forall x\ \ Smart(x) \lor (x = x)$.

Valid. In every model, every object satisfies $x = x$, so the disjunction is satisfied regardless of whether x is smart.

- (Caitlin Finnigan) 8.10 Consider a vocabulary with the following symbols:
  - Occupation(p,o): Predicate. Person p has occupation o.
  - Customer(p1, p2): Predicate. Person p1 is a customer of person p2.
  - Doctor, Surgeon, Lawyer, Actor: constants denoting occupations.
  - Emily, Joe: constants denoting people.
- Use these symbols to write these sentences in FOL.
  - Emily is either a surgeon or lawyer.
  - Joe is an actor, but he also holds another job.
  - All surgeons are doctors.
  - Joe is not a customer of any lawyer.
  - Emily has a boss who is a lawyer.
  - There is a lawyer all of whose customers are doctors.
  - Every surgeon has a lawyer.

8.10

a. $O(E, S) \vee O(E, L)$.
b. $O(J, A) \wedge \exists p \;\; p \neq A \wedge O(J, p)$.
c. $\forall p \;\; O(p, S) \;\Rightarrow\; O(p, D)$.
d. $\neg \exists p \;\; C(J, p) \wedge O(p, L)$.
e. $\exists p \;\; B(p, E) \wedge O(p, L)$.
f. $\exists p \;\; O(p, L) \wedge \forall q \;\; C(q, p) \;\Rightarrow\; O(q, D)$.
g. $\forall p \;\; O(p, S) \;\Rightarrow\; \exists q \;\; O(q, L) \wedge C(p, q)$.

- (Hugo Cheng) 8.11
  - Translate this FOL sentence into a natural English sentence:
    - $\forall x,y,l\ (SpeaksLanguage(x,l) \land SpeaksLanguage(y,l)) \Rightarrow (Understands(x,y) \land Understands(y,x))$
  - Explain why the above sentence is entailed by this sentence:
    - $\forall x,y,l\ (SpeaksLanguage(x,l) \land SpeaksLanguage(y,l)) \Rightarrow Understands(x,y)$
  - Translate the following into FOL:
    - Understanding leads to friendship.
    - Friendship is transitive.

8.11

a. People who speak the same language understand each other.

b. Suppose that an extended interpretation with $x \rightarrow A$ and $y \rightarrow B$ satisfy

$$SpeaksLanguage(x, l) \land SpeaksLanguage(y, l)$$

for some l. Then from the second sentence we can conclude $Understands(A, B)$. The extended interpretation with $x \rightarrow B$ and $y \rightarrow A$ also must satisfy

$$SpeaksLanguage(x, l) \land SpeaksLanguage(y, l) ,$$

allowing us to conclude $Understands(B, A)$. Hence, whenever the second sentence holds, the first holds.

c. Let $Understands(x, y)$ mean that x understands y, and let $Friend(x, y)$ mean that x is a friend of y.

(i) It is not completely clear if the English sentence is referring to mutual understanding and mutual friendship, but let us assume that is what is intended:
$\forall x, y \;\; Understands(x, y) \land Understands(y, x) \Rightarrow (Friend(x, y) \land Friend(y, x))$.

(ii) $\forall x, y, z \;\; Friend(x, y) \land Friend(y, z) \;\Rightarrow\; Friend(x, z)$.

- (Haojie (Tommy) Wu) 8.18 Write out the axioms required for reasoning about the Wumpus's location, using a constant symbol *Wumpus* and a predicate *At(Wumpus, Location)*. Remember that there is only one Wumpus.

8.18 We need the following sentences:

$$\forall\, s_1\ \ Smelly(s_1)\ \Leftrightarrow\ \exists\, s_2\ \ Adjacent(s_1, s_2) \land In(Wumpus, s_2)$$
$$\exists\, s_1\ \ In(Wumpus, s_1) \land \forall\, s_2\ \ (s_1 \neq s_2)\ \Rightarrow\ \neg In(Wumpus, s_2)\ .$$

- (Yui Hei (Joe) Tsui) 8.21 In Chapter6, we used equality to indicate the relationship between a variable and its value. For example, in the map coloring example, we wrote *WA = red.* In FOL, we must write *ColorOf(WA) = red*. What incorrect inference could we derive if we instead wrote sentences such as *WA = red* in FOL?

8.21 If we have $WA = red$ and $Q = red$ then we could deduce $WA = Q$, which is undesirable to both Western Australians and Queenslanders.