

Velocity Resolution with Step-Up Gearing:

As before, the speed resolution is given by the change in speed corresponding to a unity change in the count. Hence, for the pulse-counting method

$$\text{for the pulse-counting method: } \Delta\omega_c = \frac{2\pi(n+1)}{pNT} - \frac{2\pi n}{pNT} = \frac{2\pi}{pNT}$$

It follows that in the pulse-counting method, step-up gearing causes an improvement in the resolution. For the pulse-timing method:

$$\Delta\omega_t = \frac{2\pi f}{pNm} - \frac{2\pi f}{pN(m+1)} = \frac{2\pi f}{pNm(m+1)} \cong \frac{pN}{2\pi f} \omega^2$$

Note: In the pulse-timing approach, for a given speed, the resolution degrades with increasing p .

In summary, the speed resolution of an incremental encoder depends on the following factors:

1. Number of windows N
2. Count reading (sampling) period T
3. Clock frequency f
4. Speed ω
5. Gear ratio p

In particular, gearing-up has a detrimental effect on the speed resolution in the pulse-timing method, but it has a favorable effect in the pulse-counting method.

Gray Coding:

In an absolute encoder, there is a data interpretation problem associated with the straight binary code.

- Note in Table that with the straight binary code, the transition from one sector to an adjacent sector may require more than one switching of bits in the binary data.
- For example, transition from *0011 to 0100* or from *1011 to 1100* requires three bit switching, and the transition from *0111 to 1000* or from *1111 to 0000* requires four bit switching.
- If the light probes are not properly aligned along a radius of the encoder disk, or if the manufacturing error tolerances for imprinting the code pattern on the disk were high, or if environmental effects have resulted in large irregularities in the sector matrix, then the bit switching from one reading to the next may not take place simultaneously.
- This will result in ambiguous readings during the transition period. For example, in changing from *0011 to 0100*, if the *LSB switches first*, the reading becomes *0010*. In decimal form, this incorrectly indicates that the rotation was from angle 3 to angle 2, whereas, it was actually a rotation from angle 3 to angle 4.
- Such *ambiguities in data interpretation can be avoided by using a gray code*, as shown in figure for this example.
- The coded representation of the sectors is given in Table 6.2.
- Note that in the case of gray code, each adjacent transition involves only one bit switching.

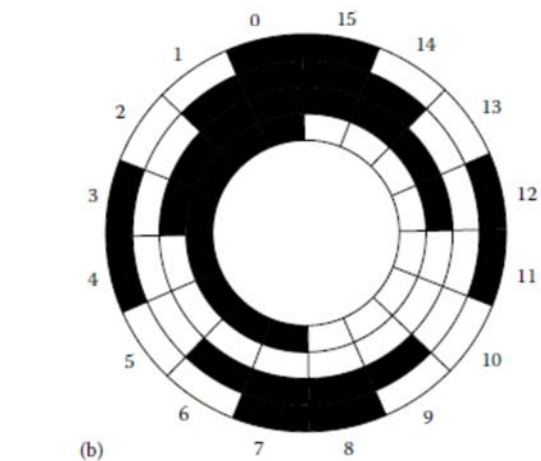
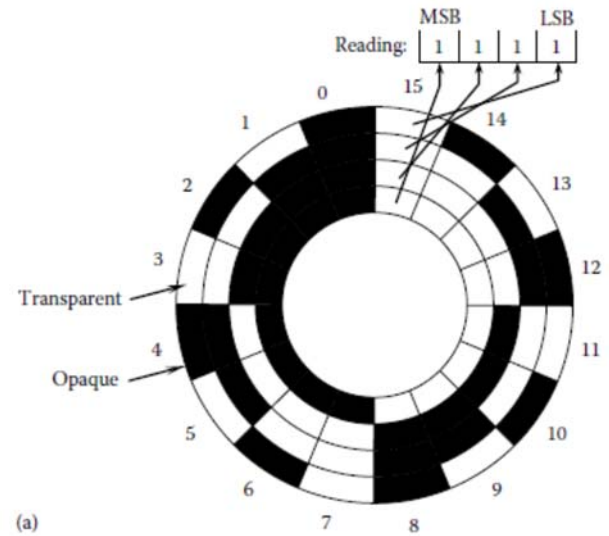


TABLE 6.2 Sector Coding for a 4-Bit Absolute Encoder

Sector Number	Straight Binary Code (MSB → LSB)	A Gray Co (MSB → L
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Encoder Error:

The primary sources of errors in shaft encoder readings can come from:

1. *Quantization error (due to digital word size limitations)*
2. *Assembly error (eccentricity of rotation, etc.)*
3. *Coupling error (gear backlash, belt slippage, loose fit, etc.)*
4. *Structural limitations (disk deformation and shaft deformation due to loading)*
5. *Manufacturing tolerances (errors from inaccurately imprinted code patterns, inexact positioning of the pick-off sensors, limitations and irregularities in signal generation and sensing hardware, etc.)*
6. *Ambient effects (vibration, temperature, light noise, humidity, dirt, smoke, etc.)*
7. *These factors can result in inexact readings of displacement and velocity and erroneous detection of the direction of motion.*

Eccentricity Error:

Eccentricity (denoted by e) of an encoder is defined as:

- *The distance between the center of rotation C of the code disk and*
- *The geometric center G of the circular code track.*
- *Nonzero eccentricity causes a measurement error known as the eccentricity error.*

The primary contributions to eccentricity are:

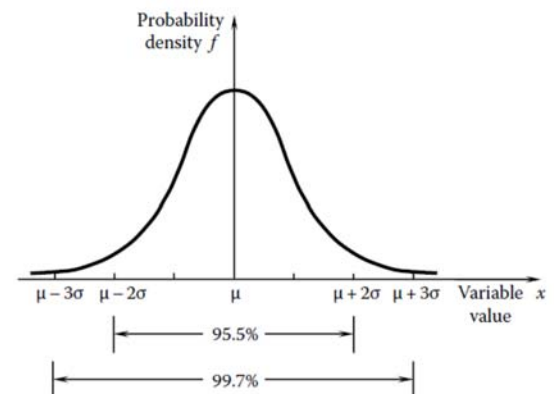
1. Shaft eccentricity (e_s): *axis of rotation is different from geometric axis.*
2. Assembly eccentricity (e_a): *center of code disk is not on the shaft axis.*
3. Track eccentricity (e_t): *center of the track is not the center of the disk.*
4. Radial play (e_p): *looseness in the assembly in radial direction.*

- All four of these parameters are random variables.
- Let their mean values be $\mu_s, \mu_a, \mu_t, \mu_p$, and
- The standard deviations be $\sigma_s, \sigma_a, \sigma_t$, and σ_p respectively.
- A very conservative upper bound for the mean value of the overall eccentricity is given by the sum of the individual absolute (i.e., considered positive) mean values. A more reasonable estimate is provided by the root-mean-square (rms) value, as given by:

$$\mu = \sqrt{\mu_s^2 + \mu_a^2 + \mu_t^2 + \mu_p^2}$$

And the standard deviation of the overall eccentricity is given by:

$$\sigma = \sqrt{\sigma_s^2 + \sigma_a^2 + \sigma_t^2 + \sigma_p^2}$$



Probability that eccentricity between:

$\mu - 2\sigma$ and $\mu + 2\sigma$: 95.5%

$\mu - 3\sigma$ and $\mu + 3\sigma$: 99.7%

Example:

The mean values and the standard deviations of the four primary contributions to eccentricity in a shaft encoder (in millimeters) are as follows:

- Shaft eccentricity = (0.1, 0.01);
- Assembly eccentricity = (0.2, 0.05);
- Track eccentricity = (0.05, 0.001);
- Radial play = (0.1, 0.02).

Estimate the overall eccentricity at a confidence level of 96%.

Solution:

Using Equation 6.23, the mean value of the overall eccentricity is estimated as the rms value of the individual means:

$$\mu = \sqrt{0.1^2 + 0.2^2 + 0.05^2 + 0.1^2} = 0.25 \text{ mm}$$

Using Equation 6.24, the standard deviation of the overall eccentricity is estimated as

$$\sigma = \sqrt{0.01^2 + 0.05^2 + 0.001^2 + 0.02^2} = 0.055 \text{ mm}$$

Now, assuming a Gaussian distribution, an estimate for the overall eccentricity at a confidence level of 96% is given by:

$$\hat{e} = 0.25 + 2 \times 0.055 = 0.36 \text{ mm}$$

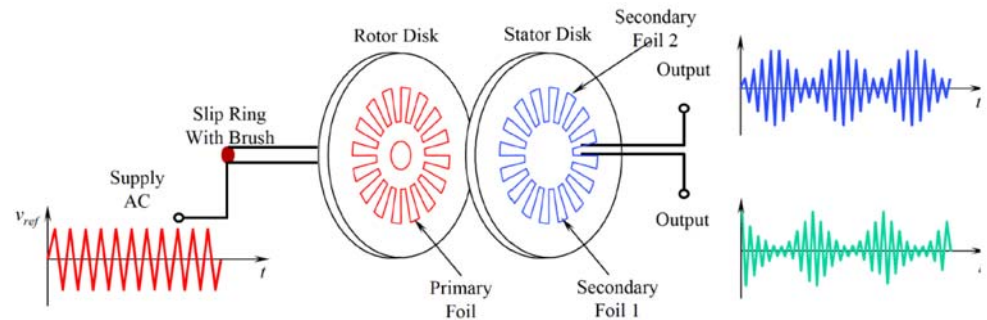
Miscellaneous Digital Transducers:

Binary Transducers:

- Digital binary transducers are two-state sensors. The information provided by such a device takes only two states (on/off, present/absent, go/no-go, high/low, etc.) which can be represented by one bit.
- For example, a limit switch is a sensor that is used in detecting whether an object has reached a particular position (or, limit), and is useful in sensing presence/absence and in object counting.
- In this sense, a limit switch is considered a digital transducer.
- Additional logic is needed *if the direction of contact* is also needed.
- Limit switches are available for both *rectilinear and angular* motions.

Digital Resolvers:

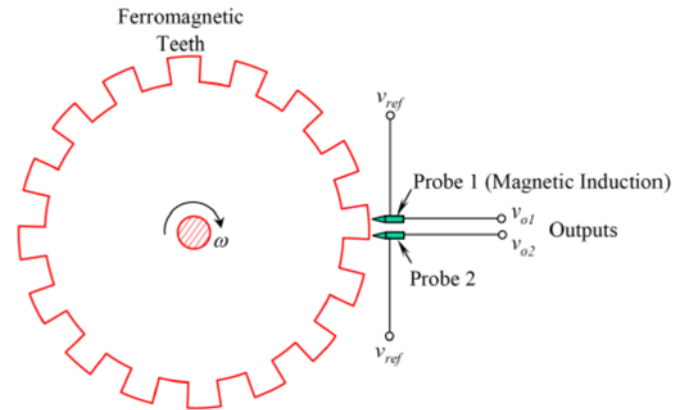
Digital resolvers, or mutual induction encoders, operate somewhat like analog resolvers, using the principle of mutual induction. They are commercially known as Inductosyns.



- A digital resolver has two disks facing each other (but not in contact), one (the stator) stationary and the other (the rotor) coupled to the rotating object whose motion is measured. The *rotor has a fine electric conductor foil imprinted on it, as shown above.*
- The printed pattern is pulse shaped, closely spaced, and connected to a high-frequency ac supply (carrier) of voltage v_{ref} .
- The stator disk has two separate printed patterns that are identical to the rotor pattern, but *one pattern on the stator is shifted by a quarter-pitch from the other pattern* (Note: pitch = spacing between two successive crests of the foil).
- The primary voltage in the rotor circuit induces voltages in the two secondary (stator) foils at the same frequency; that is, the *rotor and the stator are inductively* coupled.
- These induced voltages are quadrature signals (i.e., 90° out of phase). As the rotor turns, the level of the induced voltage changes, depending on the relative position of the foil patterns on the two disks.
- When *the foil pulse patterns coincide, the induced voltage is a maximum* (positive or negative), and when *the rotor foil pattern has a half-pitch offset from the stator foil pattern, the induced voltage in the adjacent segments cancel* each other, producing a zero output.
- Very fine resolutions (e.g., 0.0005°) may be obtained from a digital resolver, and it is usually not necessary to use step-up gearing or other techniques to improve the resolution. These *transducers are usually more expensive than optical encoders.* *The use of a slip ring and brush to supply the carrier signal may be viewed as a disadvantage.*

Digital Tachometers:

- 2 proximity sensors located at $\frac{1}{4}$ pitch from each other.
- This is a *magnetic induction, pulse tachometer of the variable-reluctance type*.
- The teeth on the wheel are made of a ferromagnetic material.
- The two magnetic-induction (and variable-reluctance) proximity probes *are placed radially facing the teeth*, at quarter-pitch apart (*pitch = tooth-to-tooth spacing*).
- When the toothed wheel rotates, the two probes generate output signals that are 90° out of phase (i.e., quadrature signals).
- One signal leads the other in one direction of rotation and lags the other in the opposite direction. *In this manner, a directional reading (i.e., velocity rather than speed) is obtained*.
- The speed is computed either by counting the pulses over a sampling period or by timing the pulse width, as in the case of an incremental encoder.
- **Advantages:** Simplicity, robustness, and low-cost.
- **Disadvantages:** Poor resolution, mechanical loading.



Alternative types of *digital tachometers use eddy current proximity probes* or capacitive proximity probes discussed in chapter-5.

- In the case of an eddy current tachometer, *the teeth of the pulsing wheel are made of (or plated with) electricity- conducting material*.
- The probe consists of an active coil connected to an ac bridge circuit excited by a radio-frequency (i.e., in the range 1–100 MHz) signal.
- The resulting magnetic field at radio frequency is modulated by the tooth-passing action. The bridge output may be demodulated and shaped to generate the pulse signal.
- In the case of a capacitive tachometer, *the toothed wheel forms one plate of the capacitor; the other plate is the probe and is kept stationary*.
- As the wheel turns, the gap width of the capacitor fluctuates. If the capacitor is excited by an ac voltage of high frequency (typically 1 MHz), a nearly pulse-modulated signal at that carrier frequency is obtained.
- This can be detected through a bridge circuit as before but using a capacitance bridge rather than an inductance bridge.
- Demodulated pulse signal generated in this manner is used in the angular velocity computation.

Moiré Fringe Displacement Sensors:

Suppose that a piece of transparent fabric is placed on another similar fabric. If one piece is moved or deformed with respect to the other, we will notice various designs of light and dark patterns (lines) in motion. Dark lines of this type are called moire fringes.

The rotation of the index plate with respect to the reference plate can be measured by sensing the orientation of the fringe lines with respect to the fixed (master or reference) gratings.

- Measurement is done by a CCD camera.
- Very high resolutions: 0.002 mm

