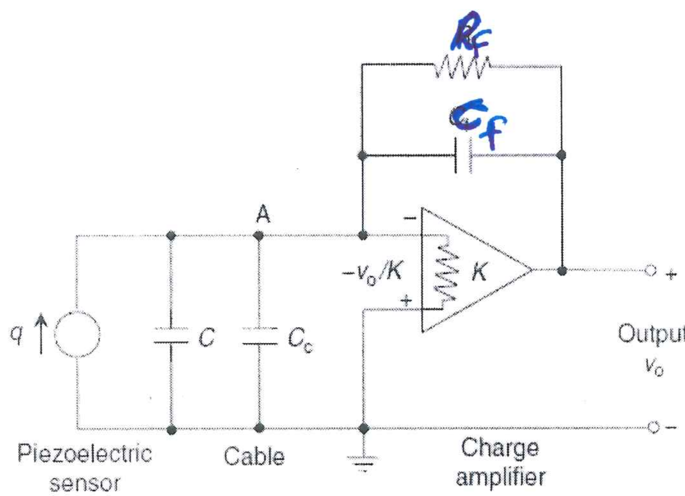
Charge Amplifier:

Piezoelectric signals cannot be read using low-impedance devices. The two primary reasons for this are:

1. High output impedance in the sensor results in small output signal levels and large loading errors.
2. The charge can quickly leak out through the load.



A charge amplifier is commonly used as the signal-conditioning device for piezoelectric sensors, in order to overcome these problems to a great extent.

- Because of impedance transformation, the impedance at the ***output of the charge amplifier becomes much smaller than the output impedance of the piezoelectric sensor***. This virtually eliminates loading error and provides a low-impedance output for purposes such as signal communication, acquisition, recording, processing, and control.
- Also, by using a charge amplifier circuit with a ***relatively large time constant, the speed of charge leakage can be decreased***.

For example, consider a piezoelectric sensor and charge amplifier combination, as represented by the circuit above. Let us examine how the rate of charge leakage is reduced by using this arrangement.

Current Balance @ point A

$$\dot{q} + C\cancel{\frac{\dot{v}_o}{K}}^{0} + C_c\cancel{\frac{\dot{v}_o}{K}}^{0} + C_f\left(\dot{v}_o + \cancel{\dot{v}_o/K}^{0}\right) + \frac{v_o + \cancel{v_o/K}^{0}}{R_f} = 0$$

K is Large ∴ Equation Reduces to $R_f C_f \frac{dv_o}{dt} + v_o = -R_f \frac{dq}{dt}$

In Laplace form $\boxed{\frac{v_o(s)}{q(s)} = \frac{-R_f s}{1 + R_f C_f s}}$; $s = j\omega$

If $\omega = 0$, $s = 0$, $\frac{v_o}{q(s)} = 0$ this can't be used for DC signals

If $s = j\omega$, is Very high, $\frac{v_o(s)}{q(s)} = -\frac{\cancel{R_f}\cancel{s}}{\cancel{R_f} C_f \cancel{s}} = -\frac{1}{C_f}$

Calibrating the charge Amplifier w.r.t. $\tau = R_f C_f$ Calibration.

System Transfer function: $G(j\omega) = \frac{j\tau\omega}{(j\omega\tau + 1)}$

Strain Gages:

- Many types of force and torque sensors are based on strain-gage measurements.
- Although strain gages measure strain, the measurements can be directly related to stress and force. Therefore, it is appropriate to discuss strain gages under force and torque sensors.
- Note, however, that strain gages may be used in a somewhat indirect manner (using auxiliary front-end elements) to measure other types of variables, including displacement, acceleration, pressure, and temperature.

Equations for Strain-Gage Measurements:

- Change of electrical resistance in material when mechanically deformed is property used in resistance-type strain gages.

Recall Resistance $R = \rho \frac{l}{A}$

l ← length of conductor; A ← x-section Area; ρ ← material resistivity

Taking Log on both sides

$$\log R = \log \rho + \log (l/A)$$

differentiating

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{d(l/A)}{l/A}$$

Fractional change in resistivity; deformation

∴ change in Resistance : change of resistivity + change in shape of Material

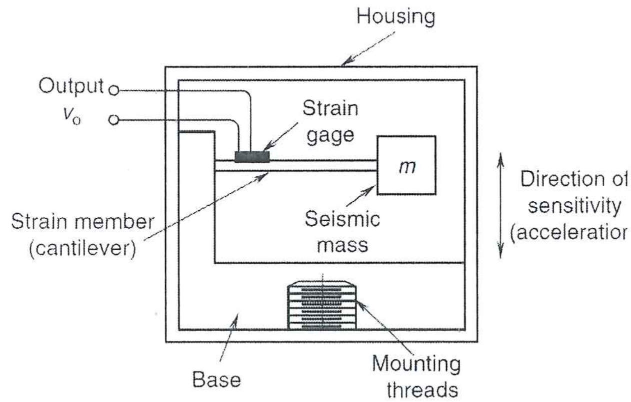
$$\therefore \quad \frac{\delta R}{R} = S_s \, \epsilon$$

ϵ ← STRAIN

S_s ← gage factor or (sensitivity of strain-gage element) 2→6 Mettalic, -- to →200 semi-conductor

Examples:

- Acceleration may be measured by first converting it into an inertia force of a suitable mass (seismic mass) element, then subjecting a cantilever (strain member) to that inertia force and, finally, measuring the strain at a high-sensitivity location of the cantilever element.



- Temperature may be measured by measuring the thermal expansion or deformation in a bimetallic element.
- Thermistors are temperature sensors made of semiconductor material whose resistance changes with temperature. Resistance temperature detectors (RTDs) operate by the same principle, except that they are made of metals, not of semiconductor material.
- Note that these temperature sensors, and the piezoelectric sensors, should not be confused with strain gages.
- Resistance strain gages are based on resistance change as a result of strain, or the piezo-resistive property of materials.

A direct way to obtain strain-gage measurement is:

- To apply a constant dc voltage across a series-connected pair of strain-gage element (of resistance R) and a suitable (complementary) resistor R_c, and
- To measure the output voltage V_0 across the strain gage under open-circuit conditions (using a voltmeter with high input impedance).
- It is known as a potentiometer circuit or ballast circuit.

Temprature Coeff.

We can define $R = R_0 + \alpha \Delta T$

$$V_0 (\text{old}) = \left(\frac{R}{R+R_c}\right) V_{ref}.$$

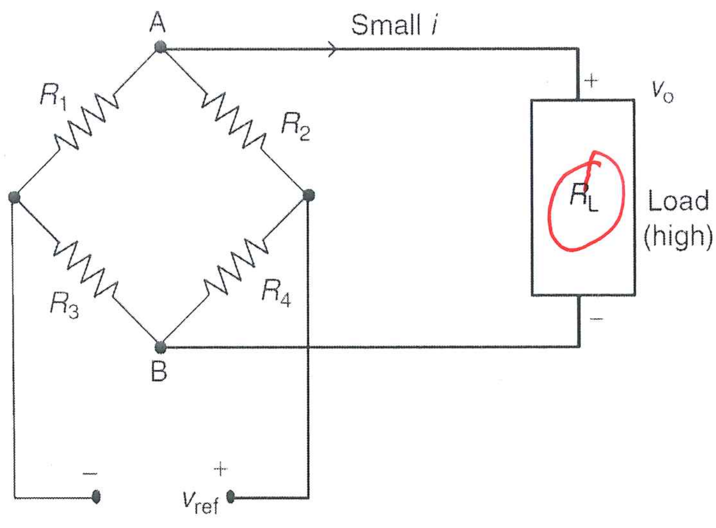
$$V_0 (\text{New}) = \left(\frac{R+\Delta R}{R+\Delta R+R_c+\Delta R}\right) V_{ref} = \left(\frac{R+\Delta R}{R+R_c+2\Delta R}\right) V_{ref}$$

Could work But Ambient temprature → strain gage R & Electrical Loading Error.

More favourable choice is wheatstone Bridge.

Bridge Sensitivity:

- Strain-gage measurements are calibrated with respect to a balanced bridge.
- When the strain gages in the bridge deform, the balance is upset.
- If one of the arms of the bridge has a variable resistor, it can be changed to restore balance.
- The amount of this change measures the amount by which the resistance of the strain gages changed, thereby measuring the applied strain.
- This is known as the *null-balance method* of strain measurement.



(Constant voltage)

Recall wheatstone bridge Equations: $v_o = \left(\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right) V_{ref} = \frac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)} V_{ref}$

When $v_o = 0$; Bridge is balanced & $\frac{R_1}{R_2} = \frac{R_3}{R_4}$; EVEN FOR SMALL R_L

* However, this Method is SLOW; Balancing bridge for each reading.

* More common method: Dynamic Readings from a strain-gage bridge.
  : Output Voltage resulting from Imbalance Caused by deformation of active-strain gages in the bridge.

$$\frac{\delta V_o}{V_{ref}} = \frac{R_2\delta R_1 - R_1\delta R_2}{(R_1+R_2)^2} - \frac{R_4\delta R_3 - R_3\delta R_4}{(R_3+R_4)^2}$$

- If all four resistors are equal (value + material), ΔR due to Ambient Conditions Cancel out. $\Rightarrow v_o$: No change
- If R_1 and R_2 & R_3 and R_4 have same Temp. Coeff: Compensation is achieved
- If R_1 changes by δR: $\frac{\delta v_o}{V_{ref}} = \frac{\delta R}{4R}$; R: strain-gage Resistance
- If $R_1 \to R+\delta R$, $R_2 \to R-\delta R$: $\frac{\delta V_o}{V_{ref}} = \frac{\delta R}{2R}$
- If $R_1, R_4 \to +\delta R$; $R_2, R_3 \to -\delta R$ $\longrightarrow \frac{\delta V_o}{V_{ref}} = \frac{\delta R}{R}$

The Bridge Constant and the Calibration Constant:

If more than one strain-gage is active, the bridge output may be expressed as:

$$\frac{\delta V_o}{V_{ref}} = K \frac{\delta R}{4R} \qquad — (1)$$

$$K = \frac{\text{bridge output in general case}}{\text{bridge output if only one strain gage is active}}$$

Bridge constant: Larger the bridge Constant Better the Sensitivity of Bridge.

Calibration Constant of a strain-gage bridge relates the strain that is measured to the output of the bridge

$$\frac{\delta V_o}{V_{ref}} = C \epsilon \qquad — (2)$$

Recall $\frac{\delta R}{R} = S_s \epsilon \qquad — (3)$

ϵ strain

S_s gage factor

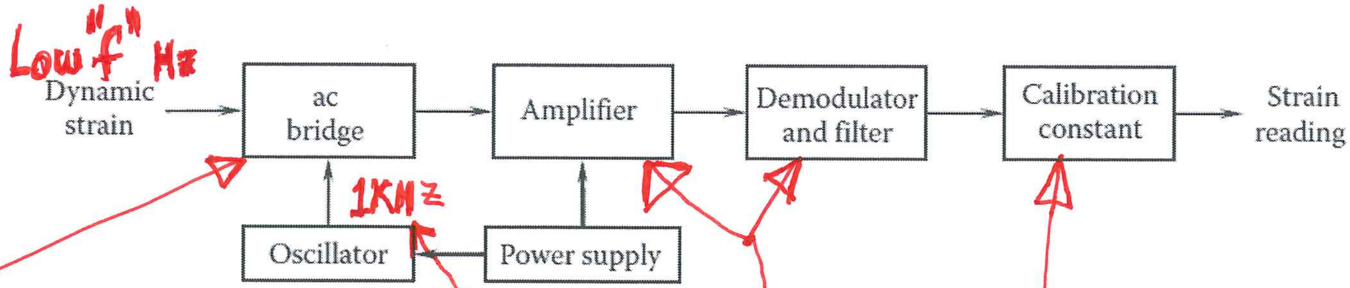
(1) + (3) $\Rightarrow$ $\frac{\delta V_o}{V_{ref}} = \frac{K}{4} S_s \epsilon . \qquad — (4)$

Now Compare (2) + (4) $\Rightarrow$ $\boxed{C = \frac{K}{4} S_s}$

$\therefore$ Calibration Constant $C = \frac{K}{4} S_s$.

Assignment: Study Example 5.2 on page 363 Textbook Version2.

Data Acquisition:



For measuring dynamic strains:

- Either the servo null-balance method or the imbalance output method should be employed (see Chapter 2).
- A schematic diagram for the imbalance output method is shown in Figure above.
- In this method, the output from the active bridge is directly measured as a voltage signal and calibrated to provide the measured strain. Figure above corresponds to the use of an ac bridge.

- The bridge is powered by an ac voltage. The supply frequency should be about 10 times the maximum frequency of interest in the dynamic strain signal (bandwidth). A supply frequency in the order of 1 kHz is typical. This signal is generated by an oscillator and is fed into the bridge. The transient component of the output from the bridge is very small (typically <1 mV and possibly a few microvolts).

- This signal has to be amplified, demodulated (especially if the signals are transient), and filtered to provide the strain reading.

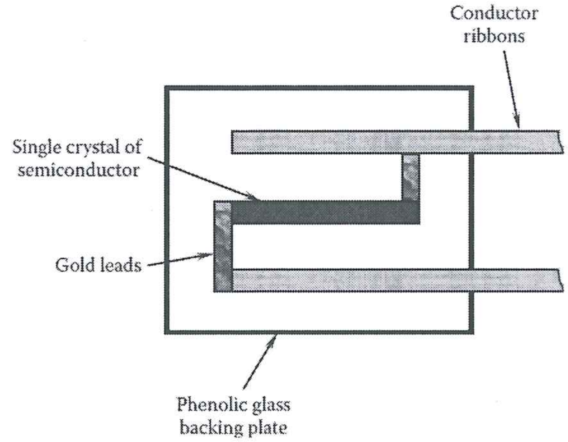
- The calibration constant of the bridge should be known in order to convert the output voltage to strain. Strain-gauge bridges powered by dc voltages are common.

- However, they have the advantages of simplicity with regard to the necessary circuitry and portability. The advantages of ac bridges include improved stability (reduced drift) and accuracy, and reduced power consumption

Accuracy Considerations:

- Foil Gauges: 50-Ω → x KΩ, Power consumption ↓ as Imp ↑; Low Heat (Bridge circuit)
  : High Range of Measurement
- Accuracy : ∝ Bridge Linearity, environmental effects (T), Mounting Tech.
  · flexibility (epoxy), Hysteresis, Hi-f- strain Measurements.

Semiconductor Strain Gauges:

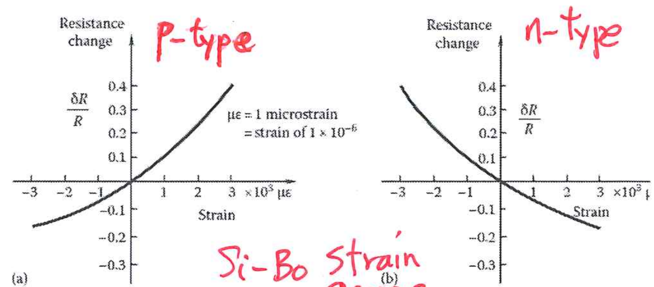


- Low-strain applications (e.g., dynamic torque measurement), the *sensitivity of foil gauges* is not adequate to produce an acceptable strain-gauge signal.
- SC strain gauges are particularly useful in such situations. The strain element of an SC strain-gauge is made of a *single crystal of piezoresistive material such as silicon, doped with a trace impurity such as boron.*
- The gauge factor (sensitivity) of an SC strain gauge is about two orders of magnitude higher than that of a metallic foil gauge (typically, 40–200), as seen for silicon, from the data given below

TABLE 5.6 Properties of Common Strain-Gauge Material

Material	Composition	Gauge Factor (Sensitivity)	Temperature Coefficient of Resistance (10^{-6}/°C)
Constantan	45% Ni, 55% Cu	2.0	15
Isoelastic	36% Ni, 52% Fe, 8% Cr, 4% (Mn, Si, Mo)	3.5	200
Karma	74% Ni, 20% Cr, 3% Fe, 3% Al	2.3	20
Monel	67% Ni, 33% Cu	1.9	2000
Silicon	p-Type	100–170	70–700
Silicon	n-Type	−140 to −100	70–700

- High resistivity - ∴ low power consumption and lower heat generation.
- Major dvantage of SC strain gauges is that they *deform elastically to fracture. Negligible mechanical hysteresis, smaller and lighter, providing less cross-sensitivity, and negligible error from mechanical loading*.
- Max-Measureable SC strain gauge is typically 0.003 m/m (i.e., 3000 με).
- Strain-gauge R - can be an order of magnitude greater for an SC strain gauge; for example, several hundred ohms for a metal foil strain gauge (typically, 120 or 350 Ω), while several thousand ohms (5000 Ω) for an SC strain gauge. Disad. associated with SC strain gauges & adv. of foil gauges.
  - *The strain–resistance relationship is more nonlinear.*
  - *Brittle and hard to mount on curvy surface*
  - *The maximum strain that can be measured is one to two orders of magnitude smaller (typically, <0.001 m/m).*
  - *Cost more and have much larger temperature sensitivity.*



non-Linear Response: $\frac{\delta R}{R} = S_1 \epsilon + S_2 \epsilon^2$.

Page 63 Notes $\frac{\delta R}{R} = S_s \epsilon$: gage factor, strain ; error $e = \frac{\delta R}{R} - \frac{\delta R}{R}\Big|_L$

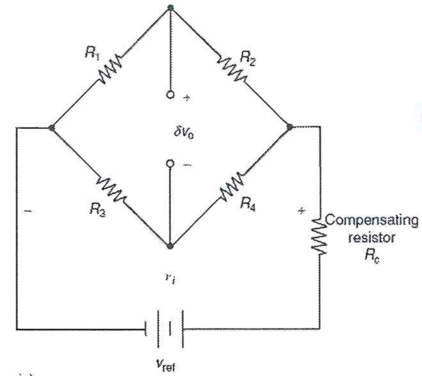
$\therefore e = S_1 \epsilon + S_2 \epsilon^2 - S_s \epsilon = (S_1 - S_s)\epsilon + S_2 \epsilon^2$; if $S_1 = S_s$; Min Error $e = S_2 \epsilon^2$

S_1 : Linear gauge factor + ; p-type, -ve for n-type.

S_2 : Non-linearity degree : + for both p + n-type.

Automatic (Self) Compensation for Temperature:

In foil gages the change in resistance due to temperature variations is typically small. Then the linear (first-order) approximation for the contribution from each arm of the bridge to the output signal, as given by Equation

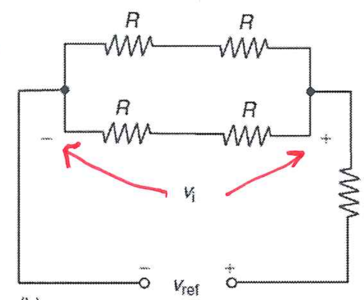


(a)

A

$$\frac{\delta V_o}{V_{ref}} = \frac{R_2\delta R_1 - R_1\delta R_2}{(R_1+R_2)^2} - \frac{R_4\delta R_3 - R_3\delta R_4}{(R_3+R_4)^2}$$

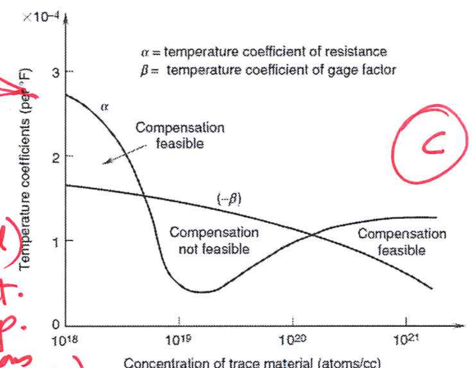
* All small changes/Temp effects cancel out due to bridge compensation



(b)

B

* In SC gages Linear approximation may not be valid
* R and S_s (strain sensitivity) or gage factor of SC strain gauge → highly dependant on concentration of strain impurity, in a non-linear manner.



C

* $R = R_0(1+\alpha\,\Delta T)$, $S_s = S_{s0}(1+\beta\,\Delta T)$ } α = Resistance coeff, $\beta = S_s$ Sensitivity coeff, ΔT = change in Temp. — Could be −ve (dep. on Material)

(A) ⇒ Constant v bridge with compensating R_C (Pre-det. to Temp. Variations to Compensate)

(B) ⇒ EQuivalent with High Load Imp.

$$V_i = \left(\frac{R}{R+R_c}\right)V_{ref}$$; Recall Page 66 Notes ; $\frac{\partial V_o}{V_{ref}} = C\epsilon$, $\frac{\partial V_o}{V_{ref}} = \frac{K}{4}S_s\epsilon$

(voltage supplied to bridge allowing for voltage drop across Resistor R_C is NOT V_{ref} but V_i

$$\frac{\partial V_o}{V_{ref}} = \left(\frac{R}{R+R_c}\right)\frac{K}{4}S_s\cdot\epsilon$$

Assume K does NOT change with temprature

0: Values before change in T.

$$\frac{R_0}{R_0+R_c}S_{s0} = \frac{R_0(1+\alpha\cdot\Delta T)}{R_0(1+\alpha\cdot\Delta T)+R_c} * S_{s0}(1+\beta\cdot\Delta T),$$

Simplifies to $R_o\beta + R_c(\alpha+\beta) = (R_o+R_c)\alpha\beta\Delta T$ — Ignore as $\alpha,\beta\Delta T$ small ⇒ ∅.

$$\therefore\ R_C = -\left[\frac{\beta}{\alpha+\beta}\right]R_0$$

⇒ COMPENSATION is possible ∵ Temp. Coeff. of Sensitivity (β) is negative.

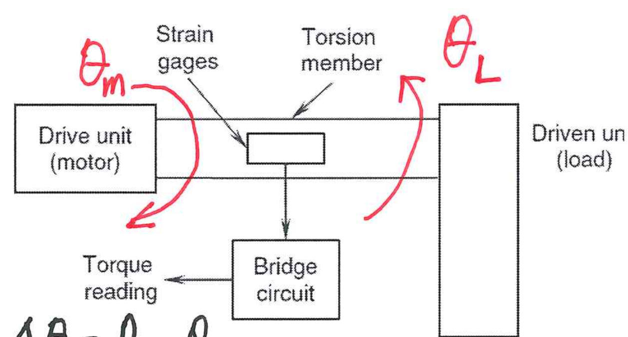
Torque Sensors:

A **torque sensor** or **torque transducer** or **torquemeter** is a device for measuring and recording the torque on a rotating system, such as an engine, crankshaft, gearbox, transmission, rotor, a bicycle crank or cap torque tester.

- Static torque is relatively easy to measure.
- Dynamic torque, on the other hand, is not easy to measure, since it generally requires transfer of some effect (electric or magnetic) from the shaft being measured to a static system.

Commonly, torque sensors or torque transducers use strain gauges applied to a rotating shaft or axle. With this method, a means to power the strain gauge bridge is necessary, as well as a means to receive the signal from the rotating shaft. This can be accomplished using slip rings, wireless telemetry, or rotary transformers. Newer types of torque transducers add conditioning electronics and an A/D converter to the rotating shaft. Stator electronics then read the digital signals and convert those signals to a high-level analog output signal, such as +/-10VDC.

Strain-Gage Torque Sensors:



$\Delta\theta = \theta_m - \theta_L$

Simple method of torque sensing is:

- To connect a torsion member between the drive unit and the (driven) load in series, as shown in diagram, and to measure the torque in the torsion member.
- If a circular shaft (solid or hollow) is used as the torsion member, the torque–strain relationship becomes relatively simple, and is given by:

$$\epsilon = \frac{r}{2GJ} T$$

Principal strain

T: Torque Transmitted through Member

J: Polar Moment of Area of x-sectional Member

G: shear Modulus of Material

r: radius.-shaft Axis.

Sheer stress is given by: $\tau = \frac{Tr}{J}$

Using Equations $\frac{\delta v_o}{v_{ref}} = C\epsilon$, $C = \frac{K}{4} S_s$ in Equation above

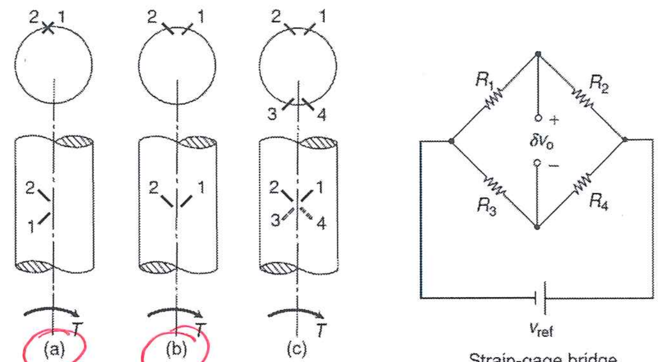
We obtain Torque T from bridge output δv_o

$$T = \frac{8GJ}{K S_s r} \cdot \frac{\delta v_o}{v_{ref}}$$

From Last Page: $T = \frac{8GJ}{kS_s r}\frac{\delta v_0}{v_{ref}}$;

- S_s is the gage factor (or sensitivity) of the strain gages and
- The bridge constant k depends on the number of active strain-gages used.

Strain gages are assumed to be mounted along a principal direction and three possible configurations are: a, b, c

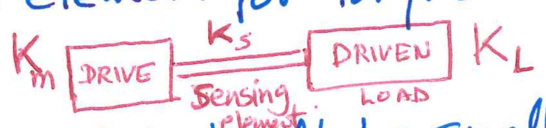


Strain-gage bridge

Configuration	(a)	(b)	(c)
Bridge constant (k)	2	2	4
Axial loads compensated	Yes	Yes	Yes
Bending loads compensated	Yes	Yes	Yes

- a, b: only 2 strain-gages, K = 2
- Both Axial and bending loads are compensated because 'R' changes by same amount.
- C: 2 pairs of gages ON TWO opposite surfaces of shaft.
- Bridge constant is doubled.
- But sensor self-compensates for axial and bending loads for first order.

Design Consideration: Torsion element for Torque Sensing



- BandWidth + Sensitivity.

From Equation: $\epsilon = \frac{r}{2GJ} T$; G.J should be small to get high ϵ

- But Tortion-Sensing element is in SERIES between drive + driven elements
- Increase in flexibility of Tortion element

$\Rightarrow$ Reduction in overall stiffness of system.

For fig above: overall stiffness K_{old} before tortion element

$$\frac{1}{K_{old}} = \frac{1}{K_m} + \frac{1}{K_L}$$

After tortion-element $\frac{1}{K_{new}} = \frac{1}{K_m} + \frac{1}{K_L} + \frac{1}{K_S}$

K_M: Motor drive Unit

K_L: Load stiffness

K_S: Torque-Sensor stiffness

$K_{new} < K_{old}$.

$\Rightarrow$ Reduction in Natural frequency and BW.

$\Rightarrow$ SLOWER RESPONSE

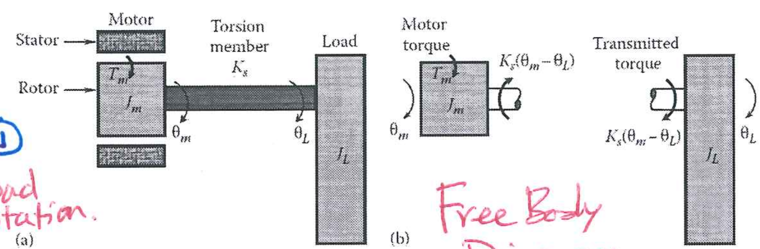
Example 5.11

Consider a rigid load, which has a polar moment of inertia J_L and driven by a motor with a rigid rotor, which has inertia J_m. A torsional member of stiffness K_s is connected between the rotor and the load, as shown in Figure 5.49a, to measure the torque transmitted to the load.

(a) Determine the transfer function between the motor torque T_m and the twist angle θ of the torsion member. What is the torsional natural frequency ω_n of the system? Discuss why the system bandwidth depends on ω_n. Show that the bandwidth can be improved by increasing K_s, by decreasing J_m, or by decreasing J_L. Give some advantages and disadvantages of introducing a gearbox at the motor output.

(b) If a torsion member of stiffness 0.5 K_s is mounted at the load end of the shaft (in series) by what percentage the original torsional bandwidth of the system (representative of the allowable operating frequency range for the torque sensor) is reduced?

Solution: Equations of Motion from free body diagram are as follows:



Free Body Diagram

Motor: $J_m\ddot{\theta}_m = T_m - K_s(\theta_m - \theta_L)$ — ①

Load: $J_L\ddot{\theta}_L = K_s(\theta_m - \theta_L)$ ②

Load Rotation

Motor Rotation

Divide ① by J_m & ② by J_L then Subtract ② from ①

$$\ddot{\theta}_m - \ddot{\theta}_L = \frac{T_m}{J_m} - \frac{K_s(\theta_m-\theta_L)}{J_m} - \frac{K_s}{J_L}(\theta_m - \theta_L) \; ; \text{ Calling } \boxed{\theta_m - \theta_L = \theta} \text{ as TWIST ANGLE}$$

∴ Equation: $\ddot{\theta} = \dfrac{T_m}{J_m} - \dfrac{K_s(\theta)}{J_m} - \dfrac{K_s}{J_L}(\theta)$ OR $\boxed{\dfrac{T_m}{J_m} = \ddot{\theta} + \left(\dfrac{1}{J_m} + \dfrac{1}{J_L}\right)K_s \cdot \theta}$ Twisting Dynamic Tortional Mode of System.

Applying Laplace: $G(s) = \dfrac{\frac{1}{J_m}}{s^2 + K_s\left(\frac{1}{J_m} + \frac{1}{J_L}\right)}$ — ① ← Characteristic EQuation of Twisting System

Tortional Twisting Natural 'f' ∴ $\omega_n = \sqrt{K_s\left(\frac{1}{J_m} + \frac{1}{J_L}\right)}$: {Rotation of Rigid body without any Twist → Natural frequency = 0}

* If θ_m or θ_L are taken as output, we can get both ω's ; If θ is taken only ω_n; other $= \emptyset$

EQ① Can be written as $G(s) = \left(\dfrac{1}{J_m}\right) \Big/ s^2 + \omega_n^2 = \dfrac{\frac{1}{J_m}}{(j\omega)^2 + \omega_n^2} = \dfrac{\frac{1}{J_m}}{\omega_n^2 - \omega^2}$ | $s = j\omega$

ω: excitation frequency

Continuing from Last Page ...

$$G(s) = \frac{\frac{1}{J_m}}{\omega_n^2 - \omega^2} ; \qquad s = j\omega$$

J_m = Inertia

J_L = Polar Moment of Inertia

K_s = stiffness.

if $\omega << \omega_n$, $G(s) = \frac{\frac{1}{J_m}}{\omega_n^2}$

implying Instantaneous response, without any dynamic delay

* BW represents the excitation frequency range 'ω' within which system responds sufficiently fast (referring to flat region of $G(s)$)

* This means as $\omega_n \uparrow$, BW (system operation) improves.

$$\omega_n = \sqrt{K_s\left(\frac{1}{J_m} + \frac{1}{J_L}\right)} ; \qquad \omega_n \uparrow \text{ as } (K_s \uparrow, J_m \downarrow, J_L \downarrow)$$

Adding a Gearbox: Equivalent Inertia ↑, Eqv. stiffness ↓ (K_s)

⇒ System BW is lowered and response will be slower

Also gearbox backlash and friction → non-linearities into system.

(b) For series-connected two tortion segments of stiffness K_s and $0.5K_s$, equivalent stiffness K_e

$$\frac{1}{K_e} = \frac{1}{K_s} + \frac{1}{0.5K_s} = \frac{3}{K_s} \quad \therefore K_e = \frac{K_s}{3}$$

From Equation: $\omega_n = \sqrt{K_s\left(\frac{1}{J_m} + \frac{1}{J_L}\right)}$; $\omega_n \propto \sqrt{K_s}$

∴ Bandwidth is reduced by a factor $\frac{1}{\sqrt{3}} \simeq 0.58$; ~ 42%