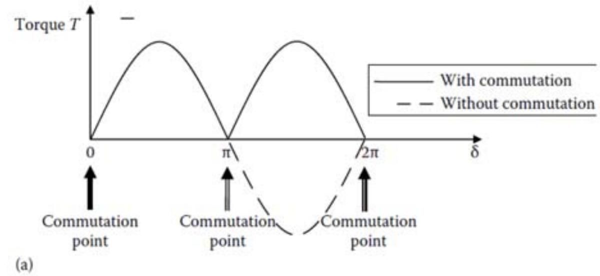
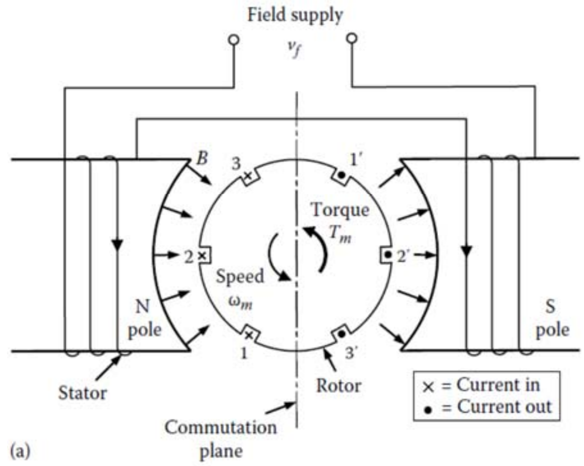
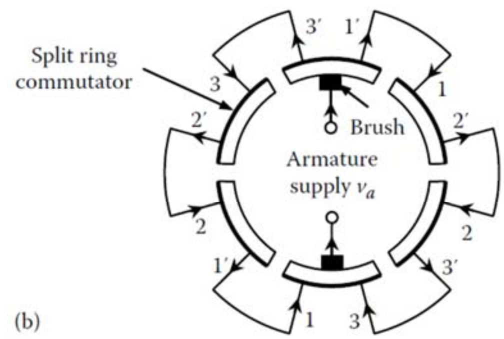
Continuing from last page:

- Suppose that the rotor rotation starts by coinciding with the commutation plane, where $\delta = 0$ or π, and the rotor rotates through an angle of 2π.
- The corresponding torque profile is shown in figure on top.

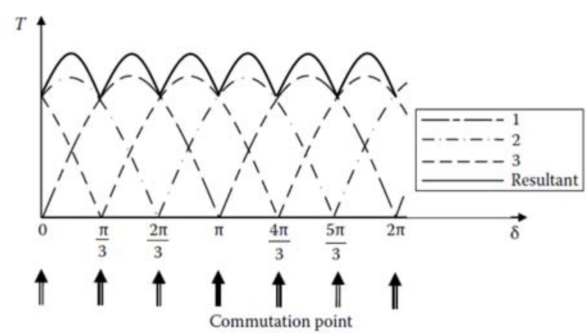


- Next suppose *that the rotor has three planar coil segments placed at 60° apart*, *and denoted by 1, 2, and 3, as in figure #2.*
- Note that current switching occurs at every 60° rotation, and in a given instant two coil segments are energized.





- Figure shows the torque profile of each coil segment and the overall torque profile due to the three-segment rotor in Figure#3.
- Note that the torque profile has improved (i.e., larger torque magnitude and smaller variation) as a result of the multiple coil segments, with shorter commutation angles.
- The *torque profile can be further improved by incorporating still more coil segments, with correspondingly shorter commutation angles*, *but the design of the split-ring and brush arrangement becomes more challenging then*.



- Hence, there is a design limitation to achieving uniform torque profiles in a dc motor.
- It should be clear from Figure#2 *that if the stator field can be made radial, then B is always perpendicular to n* and *hence sin δ becomes equal to 1*. In that case, the torque profile is uniform, under ideal conditions.

Brushless DC Motors:
Before we get into Brushless DC Motors, we should look at shortcomings of the slip-ring and brush mechanisms:

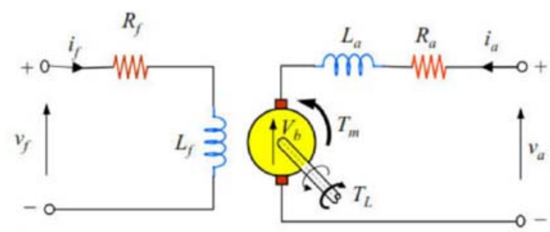
- Rapid wear out, mechanical loading,
- Heat generation due to sliding friction,
- Contact bounce,
- Excessive noise, and
- Electrical sparks with the associated dangers in hazardous (e.g., chemical) environments,
- Problems of oxidation,
- Problems in applications that require wash down (e.g., in food processing), and voltage ripples at current switching instants.
- Conventional remedies to these problems─such as the use of improved brush designs and modified brush positions to reduce sparking─are inadequate in more demanding and sophisticated applications.
- Cooling of the coils is typically needed in long-period operation of heavy-duty motors which may be achieved through forced convection of air or water.
- In addition, the required maintenance (to replace brushes and resurface the split-ring commutator) can be rather costly and time consuming.
- *Electronic communication, as used in brushless dc motors, is able to overcome these problems*.

Permanent-Magnet Motors:

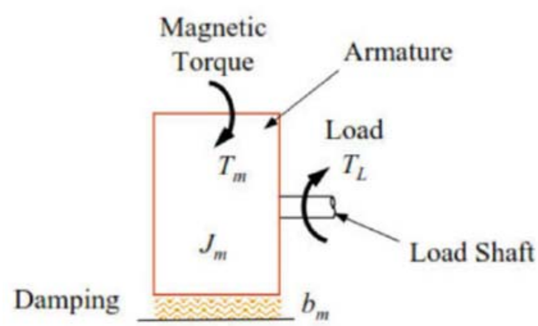
- Brushless dc motors have permanent-magnet rotors.
- Since in them the polarities of the rotor cannot be switched as the rotor crosses a commutation plane, commutation is accomplished by electronically switching the current in the stator winding segments.
- Note that this is the reverse of what is done in brushed commutation,
  - where the stator polarities are fixed and
  - The rotor polarities are switched when crossing a commutation plane.
- The stator windings of a brushless dc motor can be considered the armature windings, whereas for a brushed dc motor, rotor is the armature.
- The torque–speed characteristics of dc motors are different from those of stepper motors or ac motors.
- Brushless DC motors are commonly used for smooth torque transitions and speed control whereas, stepper motors are commonly used for stepping precision motion control.

DC Motor Equations:

- Consider a dc motor with separate windings in the stator and the rotor.
- Each coil has a resistance (R) and an inductance (L).
- When a voltage (v) is applied to the coil, a current (i) flows through the circuit, thereby generating a magnetic field.
- Forces are produced in the rotor windings, and an associated torque (T_m), which turns the rotor.
- The rotor speed (ω_m) causes the magnetic flux linkage with the rotor coil from the stator field to change at a corresponding rate, thereby generating a voltage (back e.m.f.) in the rotor coil.
- Equivalent circuits for the stator and the rotor of a conventional dc motor are shown.
- Since the field flux is proportional to the field current i_f, *we can express the magnetic torque of the motor as*: $T_m = k\, i_f i_a = k_m i_a$



Stator (Field Circuit) Rotor (Armature Circuit)



using Equation $F = Bil$ and $V_b = Blv$

Back emf generated in Armature is (ideal/steady)

$$V_b = K' i_f \omega_m = K'_m \omega_m \; ; \quad K i_f = K_m$$

i_f = field Current

i_a = Armature Current

ω_m = angular speed of Motor

k, k' : Motor Constants : dependant on dimensions, turns, μ, Reluctance etc.

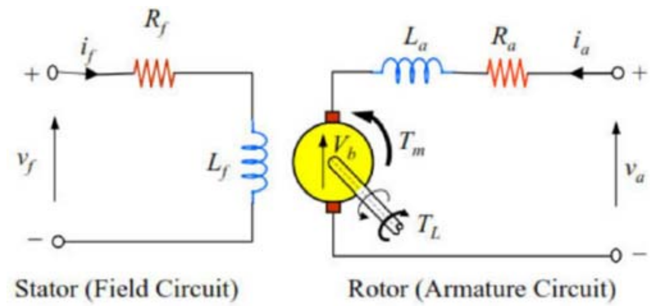
Note that: ideal Electrical to mechanical Conditions ⇒ $\boxed{T_m \times \omega_m = V_b * i_a}$

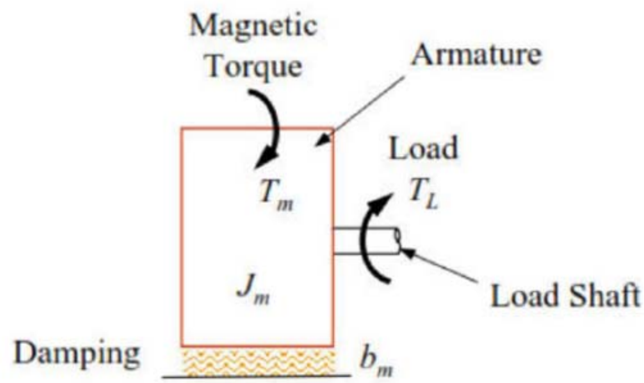
Newtons-m, $\frac{\text{rad}}{\text{sec}}$, Volts, Amperes

Under ideal Conditions $K = K'$

or $K_m = K'_m$

Obtaining equations for Field Circuit, Armature Circuit and Mechanical Dynamic:





Field Circuit Equations:

- Assuming stator Magnetic field is NOT affected by the rotor Magnetic field.

⇒ STATOR 'L' not affected by Rotor and No EDDY CURRENTS in STATOR

Stator Supply Voltage → $V_f = R_f i_f + L_f \dfrac{di_f}{dt}$

Field Winding Resistance (R_f), Field Winding Inductance (L_f)

Armature Circuit

Equation for Armature (Rotor) circuit is

Armature Supply Voltage → $V_a = R_a i_a + L_a \dfrac{di_a}{dt} + V_b$

Armature Winding Resistance (R_a)

Leakage inductance in Armature Windings (L_a)

Mechanical Dynamics:

using Newton's 2nd Law to Rotor

- Motor drives some Load, requiring Torque T_L to operate
- Frictional Resistance in Armature can be modelled as

Moment of Inertia of Rotor → $J_m \cdot \dfrac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m$

equivalent Mechanical damping constant for Rotor (b_m)

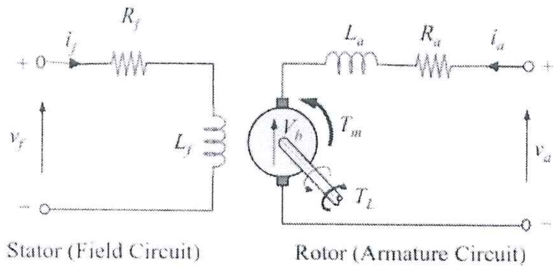
Assumptions:

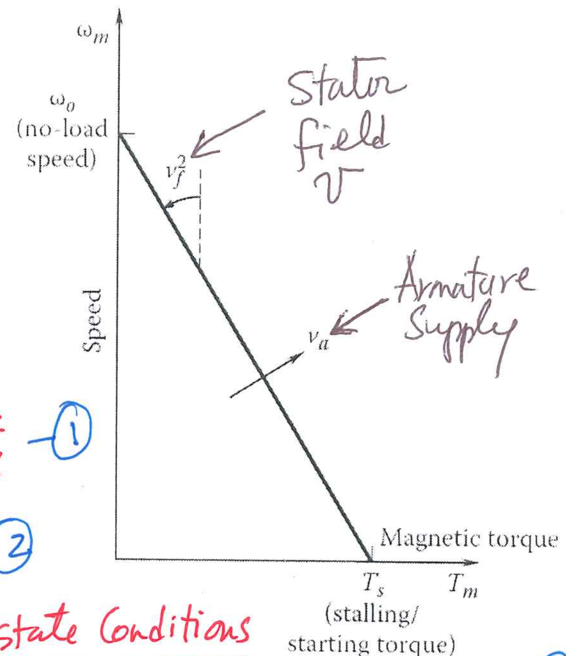
In obtaining equations for this dynamic model for the system, we have made several assumptions and approximations. In particular, we have either approximated or neglected the following factors:

1. Coulomb friction and associated *dead-band effects*.
2. Magnetic hysteresis (particularly in the stator core, but in the armature as well if not a brushless motor)
3. Magnetic saturation (in both stator and the armature)
4. Eddy current effects (laminated core reduces this effect)
5. Nonlinear constitutive *relations for magnetic induction (in which case inductance L is not constant*)
6. In split-ring and brush commutation, brush contact electrical resistance and friction, finite width contact of brushes, and other types of noise and nonlinearities
7. The effect of the rotor magnetic flux (armature flux) on the stator magnetic flux (field flux)

Steady-State Characteristics:

- In selecting a motor for a given application, its steady-state characteristics are a major determining factor.
- Steady-state torque–speed curves are employed for this purpose.
- The rationale is that, if the motor is able to meet the steady-state operating requirements, with some design conservatism, *it should be able to tolerate some deviations under transient conditions of short duration.*
- In the separately excited case shown in top figure, where the armature circuit and field circuit are excited by separate and independent voltage sources, it can be shown that the steady-state torque–speed curve is a straight line.





starting With Last page $V_f = R_f i_f + L_f \frac{di_f}{dt}$ — (1)

and

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + v_b \quad — (2)$$

Take derivative and Set $= \emptyset$. $\Rightarrow$ steady state Conditions

Note i_f is constant for fixed Supply v_f

Also previously $T_m = \underbrace{k i_f}\, i_a = k_m i_a$ (3) and $V_b = \underbrace{k' i_f}\, \omega_m = k'_m \omega_m$ (4)

Substituting (4) & (5) into (2) We get $\boxed{v_a = \frac{R_a}{k i_f} \cdot T_m + k' i_f \omega_m}$

Under steady state in field circuit from (1). $V_f = i_f R_f$

Torque speed may be expressed as:

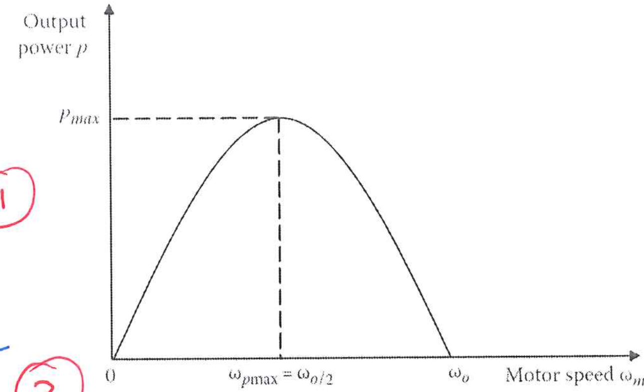
$$\omega_m + \frac{R_a R_f^2}{k^2 v_f^2} T_m = \frac{R_f v_a}{k v_f} \quad \text{or} \quad \omega_m + \frac{R_a}{K_m^2} T_m = \frac{v_a}{K_m}$$

$\rightarrow$ V_a, V_f : Constant $\longrightarrow$ defining T_s (Stalling/starting) Torque, ω_0 (no-Load speed & no-damping $\Downarrow$ $b_m = 0$)

$$\frac{\omega_m}{\omega_0} + \frac{T_m}{T_s} = 1$$

Read Example 9.1 Pg. 667 text

Output Power:



Output Power of a motor is given by

$$P = T_m \omega_m \quad (1)$$

Magnetic Torque

using $\frac{\omega_m}{\omega_0} + \frac{T_m}{T_S} = 1 \quad (2)$

Motor Angular speed

no load speed

stalling Torque or starting

Substitute (2) in (1)

$$P = T_s\left(1 - \frac{\omega_m}{\omega_0}\right)\omega_m \quad (3)$$

shown in graphic form as quadratic shape

Pt. of Max Power: differentiate (3) w.r.t speed & = 0.

$$\therefore \frac{dP}{d\omega_m} = T_s\left(1 - \frac{\omega_m}{\omega_0}\right) - \frac{T_s}{\omega_0}\omega_m = T_s\left(1 - 2\frac{\omega_m}{\omega_0}\right) = 0.$$

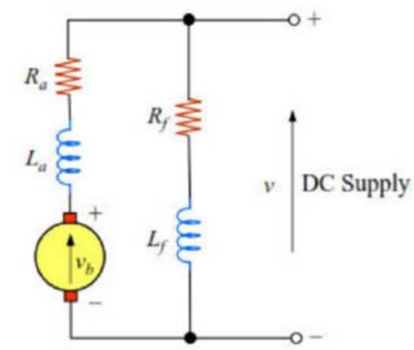
$$\therefore \omega_{p(max)} = \frac{\omega_0}{2}$$; Max Power is given by $\frac{1}{2}$ no-load speed.

& The max Power value is

$$P_{max} = \frac{1}{4} T_s \omega_0$$

Combined Excitation of Motor Windings:

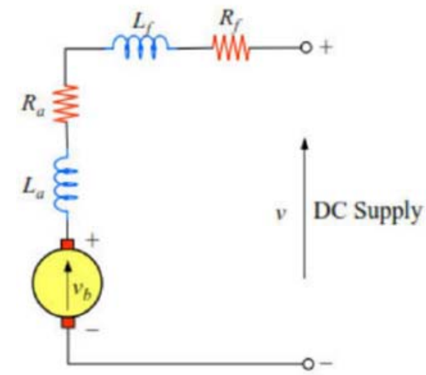
The shape of the steady-state speed–torque curve will change if a common voltage supply is used to excite both *the field windings* and *the armature windings*. Here, the two windings have to be connected together.



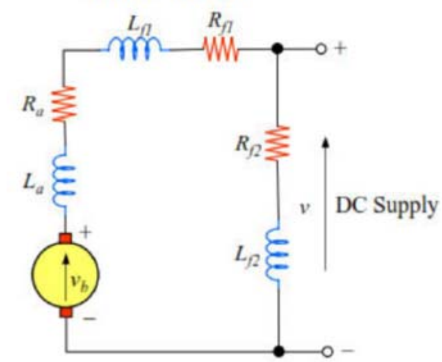
Shunt Motor

There are three common ways the windings of the rotor and the stator are connected.

1. Shunt-wound motor
2. Series-wound motor
3. Compound-wound motor



Series Motor



Compound Motor

- *In a shunt-wound motor*, the armature windings and the field windings are connected in parallel.
- *In the series-wound motor*, they are connected in series.
- *In the compound-wound motor*, part of the field windings is connected with the armature windings in series and *the other part is connected in parallel.*
- Note that in a shunt-wound motor at steady state, the back e.m.f. v_b depends directly on the supply voltage.
- Since the *back e.m.f. v_b is proportional to the speed*, it follows that *speed controllability is good with the shunt-wound configuration.*
- In a series-wound motor,
  - *The relation between v_b and the supply voltage is coupled through both the armature windings and the field windings*.
  - Hence its *speed controllability is relatively poor*.
  - But in this case, a relatively large current flows through both windings at low speeds of the motor (when the back e.m.f. is small), *giving a higher starting torque*.
  - Also, the *operation is approximately at constant power in this case*. These properties are summarized in Table.
  - Since both speed controllability and higher starting torque are desirable characteristics, *compound-wound motors* are used to obtain a performance in between the two extremes.

TABLE 9.1 Influence of the Winding Configuration on the Steady-State Characteristics of a DC Motor

DC Motor Type	Field Coil Resistance	Speed Controllability	Starting Torqu
Shunt-wound	High	Good	Average
Series-wound	Low	Poor	High
Compound-wound	Parallel high, series low	Average	Average

Speed Regulation:

Variation in the operating speed of a motor due to changes in the external load is measured by the percentage speed regulation. Specifically,

$$Percent\ Speed\ Regulation = \frac{\omega_o - \omega_f}{\omega_f} * 100\%$$

$$where, \boldsymbol{\omega_o}\ is\ the\ no-load\ speed\ ;\ \boldsymbol{\omega_f}\ is\ the\ full-load\ speed$$

- This is a measure of the speed stability of a motor;
  - The *smaller the percentage speed regulation, the more stable the operating speed under varying load conditions* (particularly in the presence of load disturbances).
  - In the shunt-wound configuration, *the back e.m.f.*, and hence the rotating speed, depends:
    - Directly on the supply voltage.
    - Consequently, the armature current and the related motor torque have virtually no effect on the speed.
    - The *percentage speed regulation is relatively small for shunt-wound motors*, resulting in improved speed stability.

HWK - Read Example 9.3

Electrical Damping Constant:

Newton's second law governs the dynamic response of a motor. Looking at previous equation:

Motor Moment of Inertia → $J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m$ Mechanical damping Constant

- b_m denotes the mechanical (viscous) damping constant and represents mechanical dissipation of energy.
- As is intuitively clear, mechanical damping torque opposes motion - hence the negative sign in the $b_m\omega_m$ term in equation.
- The *magnetic torque* T_m of the motor is also dependent on speed ω_m.
- In particular, the back e.m.f., which is governed by ω_m, produces a magnetic field, which tends to oppose the motion of the motor rotor.
- This acts as a damper, and the corresponding damping constant is given by

$$b_e = -\frac{\partial T_m}{\partial \omega_m}$$; Electrical damping Constant