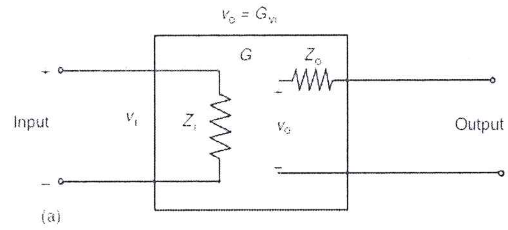
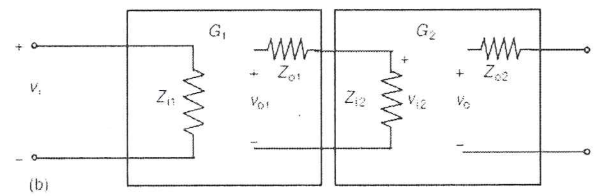
Z_0 and Z_i can be represented as shown in the diagram.

- Note that v_0 is the open-circuit output voltage.
- When a load is connected at the output port, the voltage across the load will be different from v_0 because this is caused by the presence of a current through Z_0





In frequency domain V_o and V_i are shown by Fourier Spectrum

We can get a Transfer function $G(jw)$ under No-load conditions as:

$$V_o = G V_i$$

Using two devices connected in cascade fashion

$$V_{o1} = G_1 V_i$$

$$V_{12} = \frac{Z_{i2}}{Z_{01} + Z_{i2}} V_{o1} \text{ and } V_o = G_2 V_{i2}$$

Combining Relation to get overall input/output

$$V_o = \frac{Z_{i2}}{Z_{01} + Z_{i2}} \underbrace{G_2 G_1}_{} V_i$$

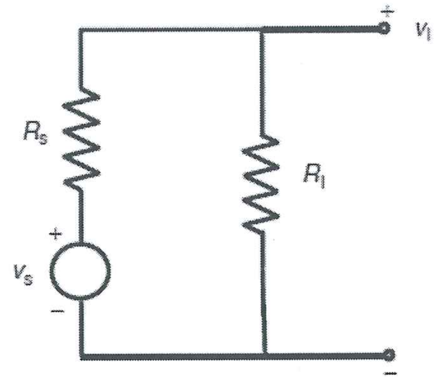
distortion from Ideal

Output Z of 1st device

$$\frac{Z_{i2}}{Z_{o1} + Z_{i2}} = \frac{1}{(Z_{01}/Z_{12}) + 1} \quad ; \text{ If } \frac{Z_{01}}{Z_{12}} \ll 1 \text{ ; deviation is insignif.}$$

Cascading is good iff $Z_{01} \ll Z_{12}$ Input Imp of Second device

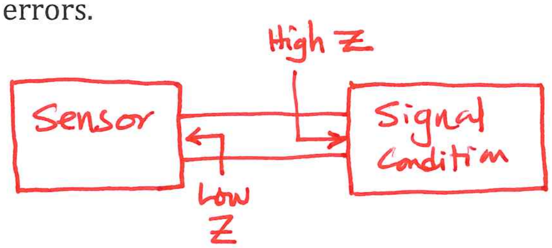
Loading Effect and Impedance Matching:

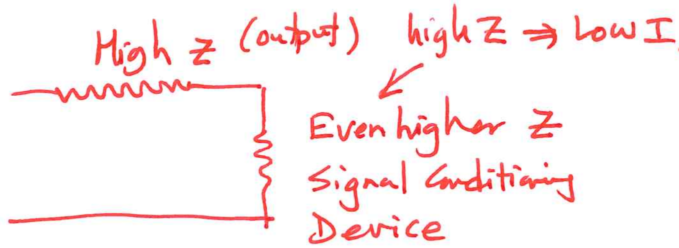


- When two electrical components are interconnected, current (and energy) flows between the two components and changes the original (unconnected) conditions. *This is known as the (electrical) loading effect, and it has to be minimized.*
- At the same time, adequate power and current would be needed for signal communication, conditioning, display, and so on.
- Both situations can be accommodated through proper *matching of impedances* when the two components are connected. Usually, an impedance-matching amplifier (i.e., an impedance transformer) would be needed between the two components.

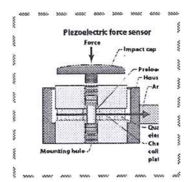
From the analysis given in the preceding section:

- The signal-conditioning circuitry should have a considerably large input impedance in comparison with the output impedance of the sensor–transducer unit to reduce loading errors.





Example: Problem is quite serious in measuring devices such as piezoelectric sensors, which have very high output impedances. A **piezoelectric** sensor is a device that uses the **piezoelectric** effect, to measure changes in pressure, acceleration, temperature, strain, or force by converting them to an electrical charge.

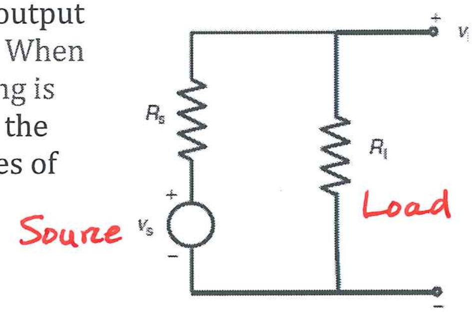


- *In such cases, the input impedance of the signal-conditioning unit might be inadequate to reduce loading effects;*
- *Also, the output signal level of these high impedance sensors is quite low for signal transmission, processing, actuation, and control.*
- *The solution for this problem is to introduce several stages of amplifier circuitry between the output of the first hardware unit (e.g., sensor) and the input of the second hardware unit (e.g., data acquisition unit).*
- *The first stage of such an interfacing device is typically an impedance-matching amplifier that has high input impedance, low output impedance, and almost unity gain.*
- *The last stage is typically a stable high-gain amplifier stage to step up the signal level. Impedance-matching amplifiers are, in fact, op-amps with feedback.*

When connecting a device to a signal source, loading problems can be reduced by making sure that the device has a high input impedance.

Unfortunately, this will also reduce the level (amplitude, power) of the signal received by the device. In fact, a high impedance device may reflect back some harmonics of the source signal. A termination resistance might be connected in parallel with the device to reduce this problem.

In many data acquisition systems, output impedance of the output amplifier is made equal to the transmission line impedance. When maximum power amplification is desired, conjugate matching is recommended. In this case, input and output impedances of the matching amplifier are made equal to the complex conjugates of the source and load impedances, respectively.



Voltage across the load is:

$$v_L = i_L R_L = v_s \cdot \frac{R_L}{R_L + R_S}$$

Power absorbed is: $i_L v_L = i_L (i_L R_L)$ Vs

$$= i_L^2 R_L$$

$$= \left(\frac{V_s}{R_L + R_S}\right)^2 \cdot R_L$$

$$\therefore \quad P = \frac{v_s^2 \cdot R_L}{(R_L + R_S)^2}$$

For Max Power $\quad \frac{dP_L}{dR_L} = 0$

$$\therefore \frac{\left[(R_S+R_L)^2 - 2\,R_L\,(1+0)\,(R_S+R_L)\right] v_s^2}{(R_S+R_L)^2} = 0$$

$$\therefore \quad (R_S+R_L)^2 - 2\,R_L(R_S+R_L) = 0$$

$$(R_S+R_L)^{\cancel{2}} = 2\,R_L\cancel{(R_S+R_L)} = 0$$

$$R_S + R_L = 2\,R_L$$

$$\therefore \quad \boxed{R_S = R_L}$$

Using Complex Impedance:

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\Gamma = \frac{Z_L}{Z_0} \text{ or } \frac{Z_0}{Z_L} \text{ depending on } Z_L > \text{ or } < Z_0$$

Standing Wave Ratio:

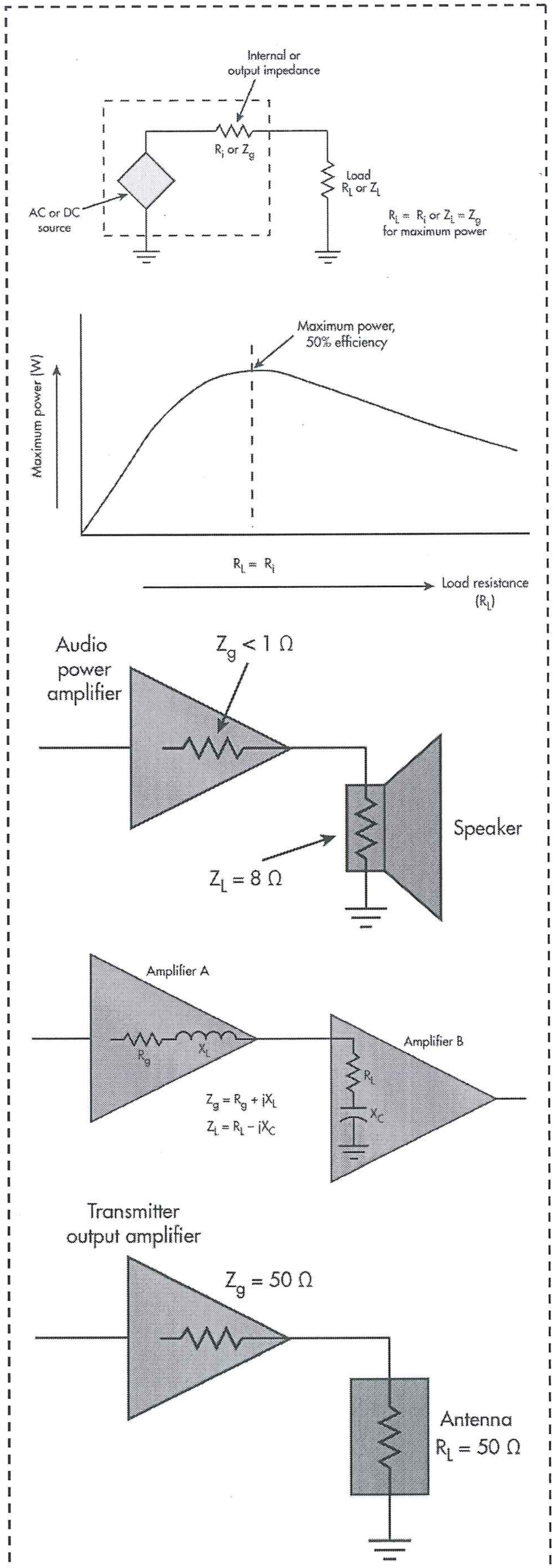
$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\Gamma = \frac{SWR-1}{SWR+1}$$

ideally $\Gamma = 0$, $SWR = 1$

No Signal Reflection

$$SWR = 1 \Rightarrow Z_L = Z_0 = Z_g.$$



Modulation:

$x(t)$ = Modulating Signal (base band)

$x_c(t)$ = High frequency Carrier

$$X_a(t) = x(t) \cdot x_c(t)$$

$$x(t) = a_c \cos(2\pi f_c t)$$

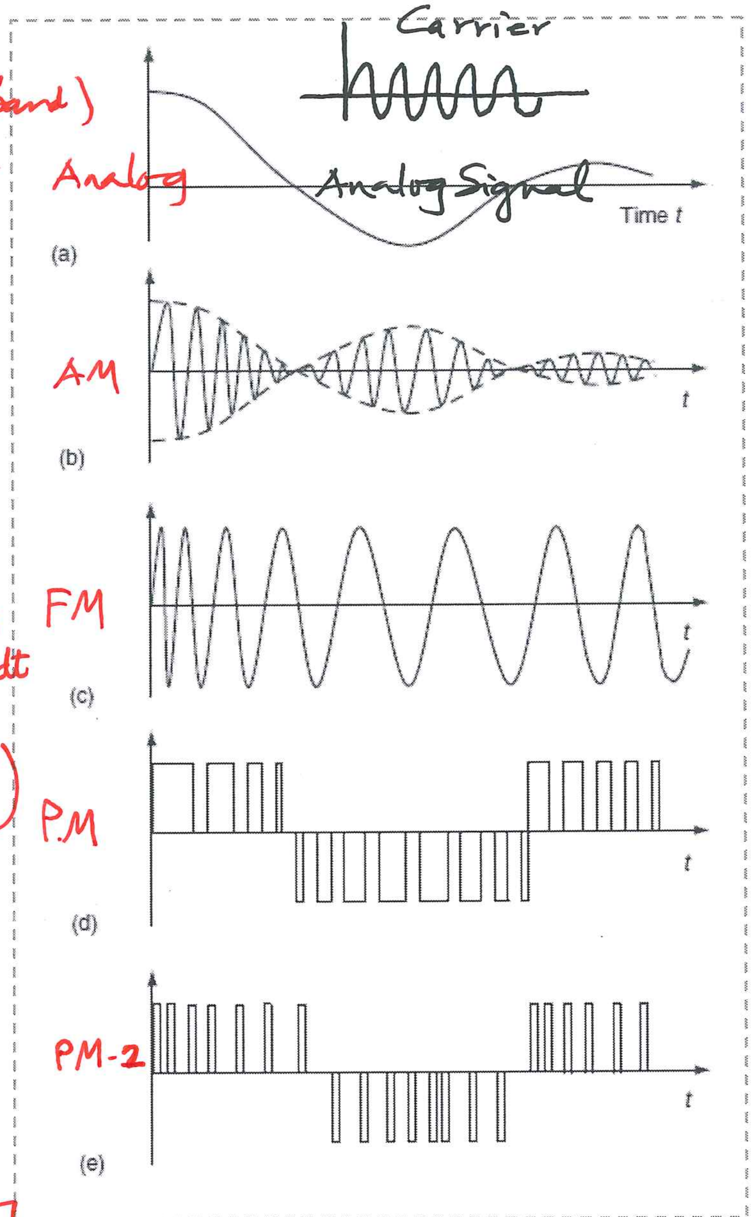
Fourier integral transform

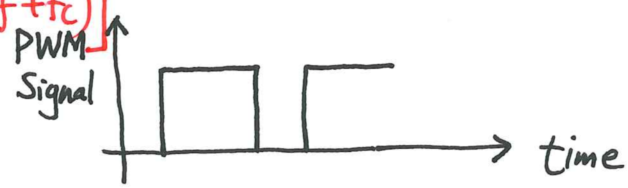
$$X_a(f) = a_c \int_{-\infty}^{\infty} x(t) \cos(2\pi f_c t)\, e^{-j2\pi f t} \cdot dt$$

$$\cos(2\pi f_c t) = \frac{1}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$$

$$X_a(f) = \frac{1}{2} a_c \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi (f-f_c)t} \cdot dt + \frac{1}{2} a_c \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi (f+f_c)t} \cdot dt$$

or $$X_a(f) = \frac{1}{2} a_c \left[X(f-f_c) + X(f+f_c)\right]$$





Duty Cycle $= \frac{\Delta T}{T} * 100\%$

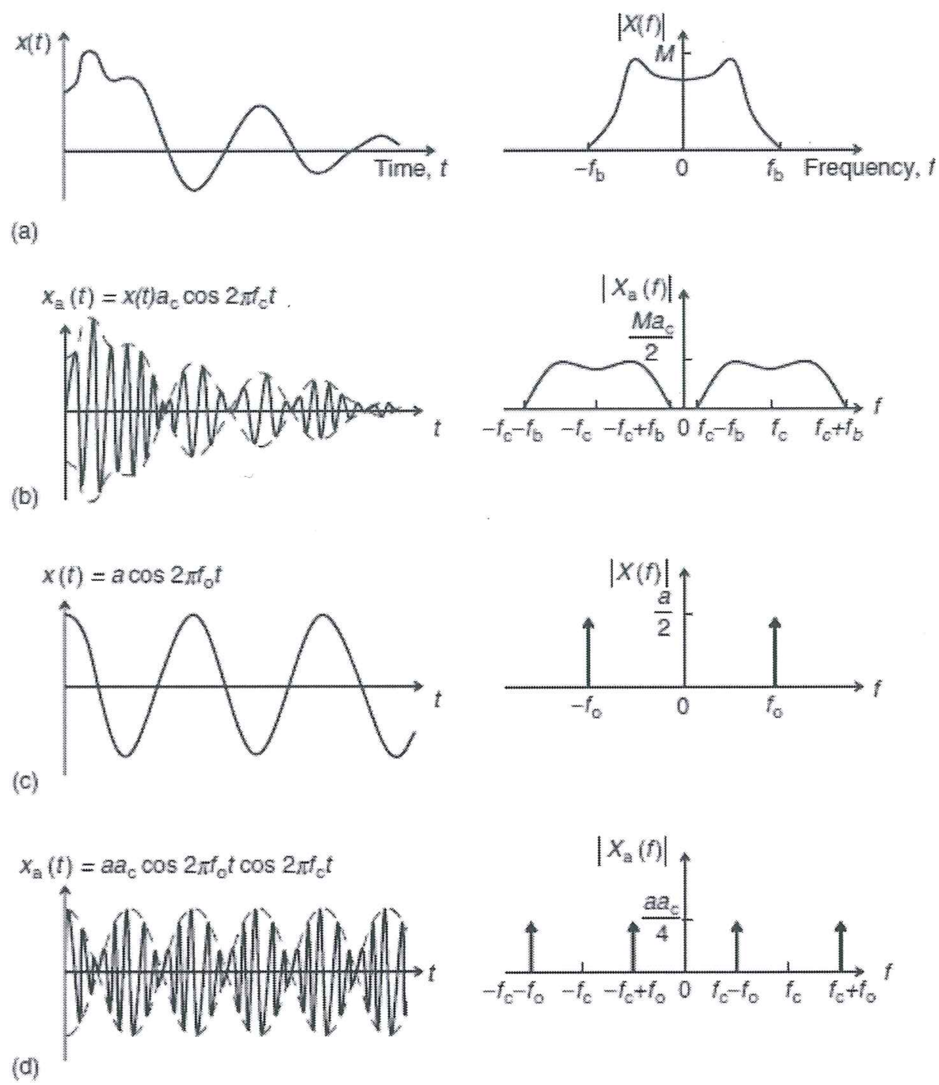
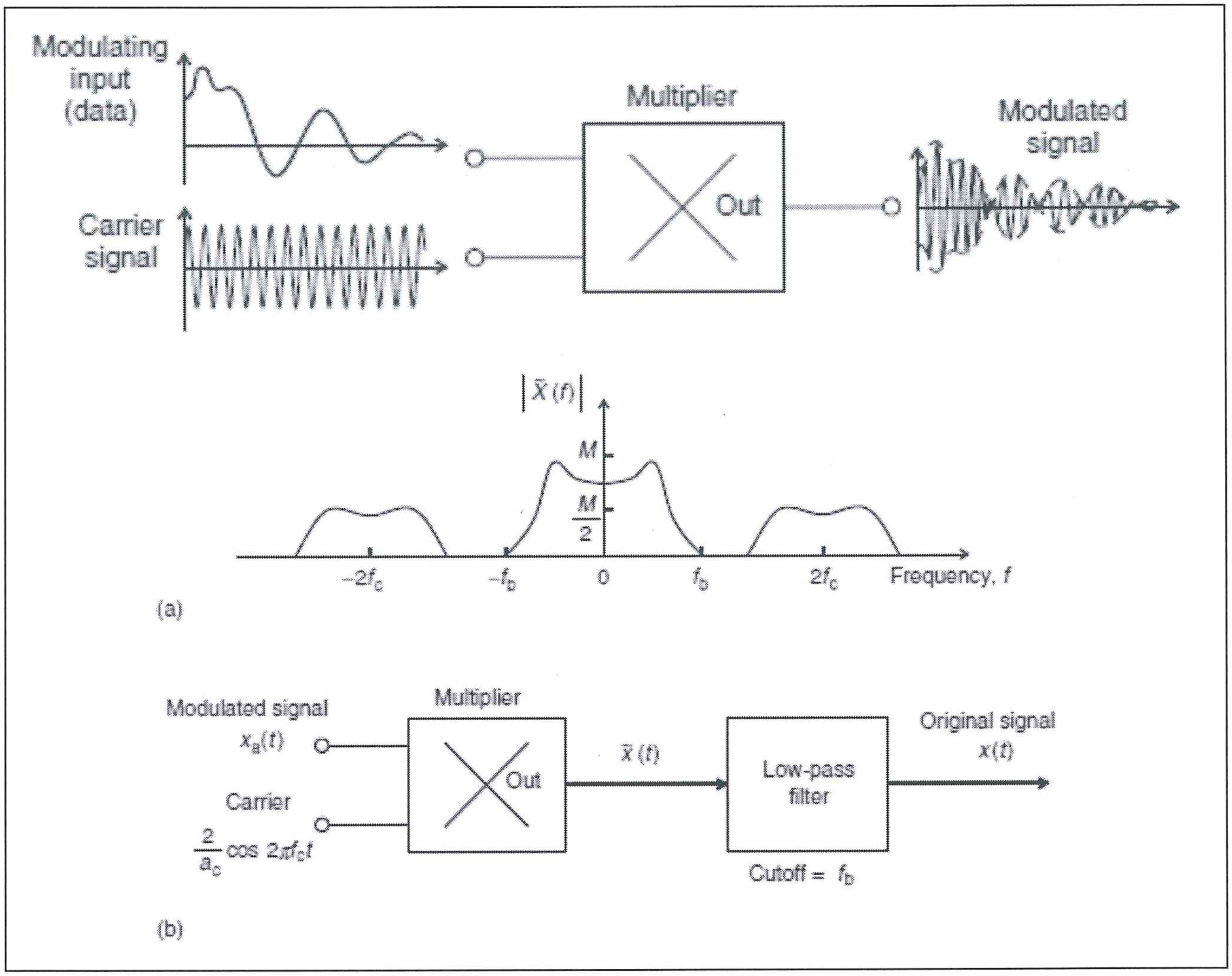


FIGURE 2.22
Illustration of the modulation theorem. (a) A transient data signal and its Fourier spectrum magnitude. (b) Amplitude-modulated signal and its Fourier spectrum magnitude. (c) A sinusoidal data signal. (d) Amplitude modulation by a sinusoidal signal.

Modulation and De-Modulation:



Consider the modulated signal above:

If Signal is Multiplied with Sinusoidal Carrier $2/a_c \cos 2\pi f_c t$

$$\tilde{x}(t) = \frac{2}{a_c} x_a(t) \cos 2\pi f_c t$$

Applying $X_a(f) = \frac{1}{2} a_c (X(f-f_c) + X(f+f_c)$, we get: (fig A).

$$\tilde{x}(f) = \frac{1}{2}\frac{2}{a_c}\left[\frac{1}{2} a_c \{X(f-2f_c) + X(f) + \frac{1}{2} a_c \{X(f) + X(f+2f_c)\}\right]$$

or

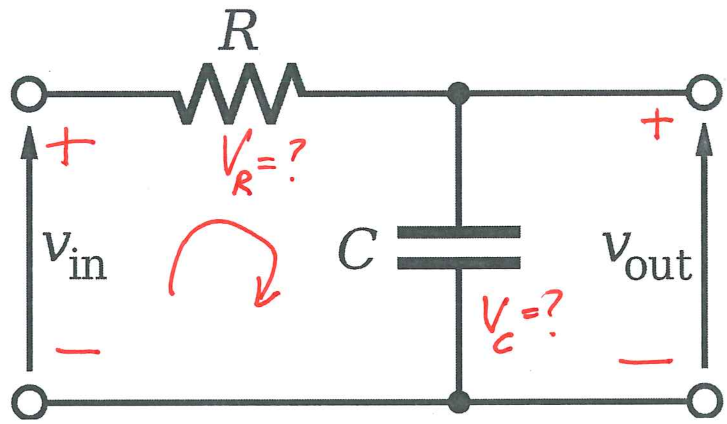
$$\hat{x}(f) = X(f) + \frac{1}{2} X(f-2f_c) + \frac{1}{2}(f+2f_c)$$

Original

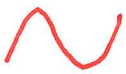
Left Side Band   Right Side Band.

Low pass Filter this to Remove these Bands.

Low Pass Filter with a cut-off:



Given a voltage @ input

 or 

step   continuous Wave.

For Resistor Load. ohm's Law $V = IR$

$$V_R = I_R R$$

Across Capacitor Current $I = c\frac{dv}{dt}$.

V_c across Capacitor

$$\therefore I_c = C\frac{dV_c}{dt}$$

using Voltage Load $V_R \rightarrow$ Resistor, $V_c \rightarrow$ Capacitor

$$\underbrace{I_R = I_c} \quad \text{and} \quad \underbrace{V_{in} = V_R + V_c}$$

$$\frac{V_R}{R} = c\frac{dV_c}{dt} \quad \text{and using} \quad V_R = V_{in} - V_c$$

sub

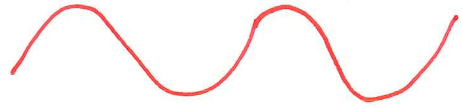
$$\frac{V_{in} - V_c}{R} = \frac{c\,dV_c}{dt}$$; taking $-\frac{V_c}{R}$ to Right.

$$\frac{c\,dV_c}{dt} + \frac{V_c}{R} = \frac{V_{in}}{R}$$

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_{in}}{RC}$$; But $V_c = V_{out}$

$$\boxed{\frac{dv_o}{dt} + \frac{1}{RC}v_o = \frac{1}{RC}V_{in}}$$

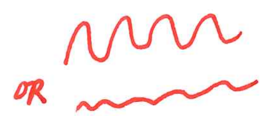
Let's say $V_{in} = V^* \sin \omega t$



$$V_o = A \cos \omega t + B \sin \omega t .$$

$$= -\frac{\omega RC}{1+(\omega RC)^2} V^* . \cos \omega t + \frac{1}{1+(\omega RC)^2} V^* \sin \omega t$$

Look @ Magnitude of output.

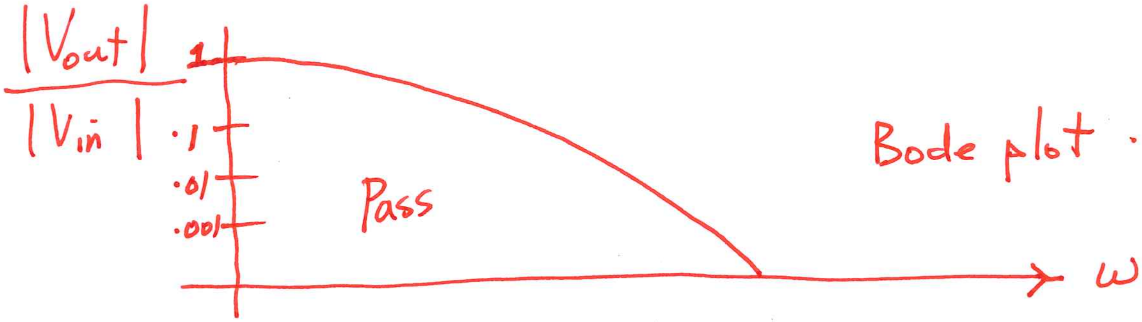


$$|V_{out}| = \frac{1}{1+\omega^2 (RC)^2} V^* = \frac{1}{1+\omega^2(RC)^2} |V_{in}| \; ;$$

Also write $\left|\frac{V_o}{V_i}\right| = \frac{1}{\tau s+1}$

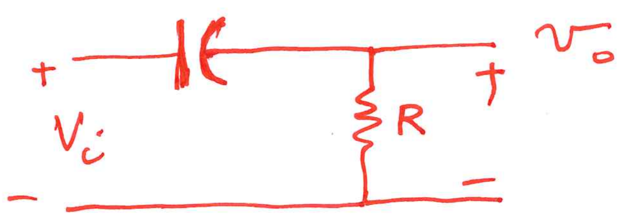
check ω is small; V_o is almost 1.

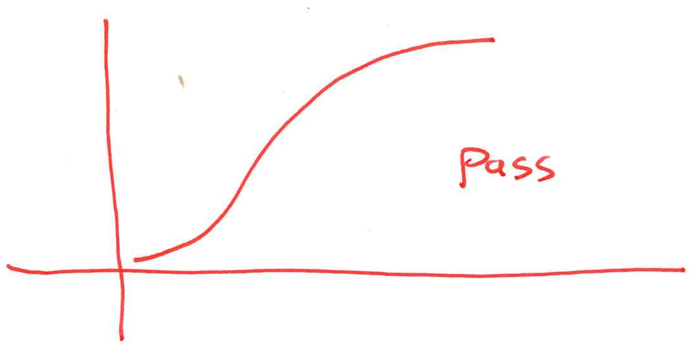
ω is high; V_o is almost $\emptyset$.

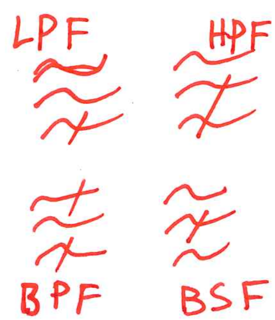


Bode plot.

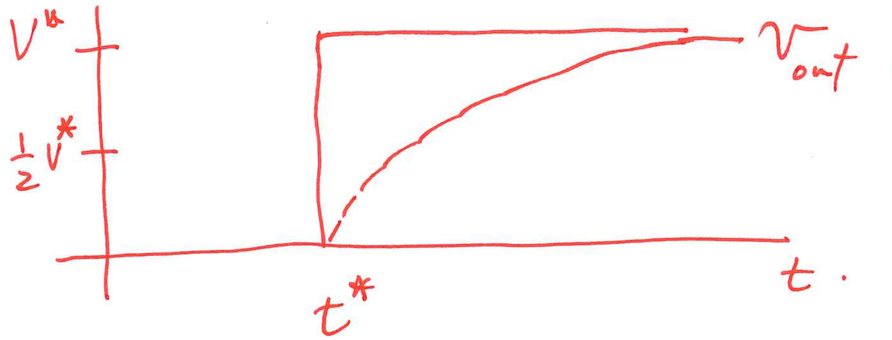
High Pass:







What if you applied a Step Function to this filter? What would V_o look like?



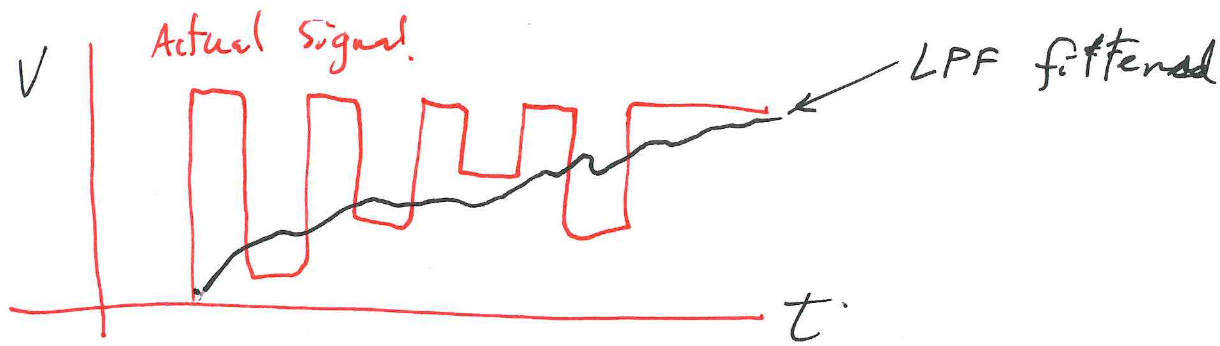
$$V_o = V^*\left[1 - e^{-\frac{1}{RC}(t-t^*)}\right]$$

If we wanted to see how long it took to get to $\frac{1}{2}V^*$, we do the following.

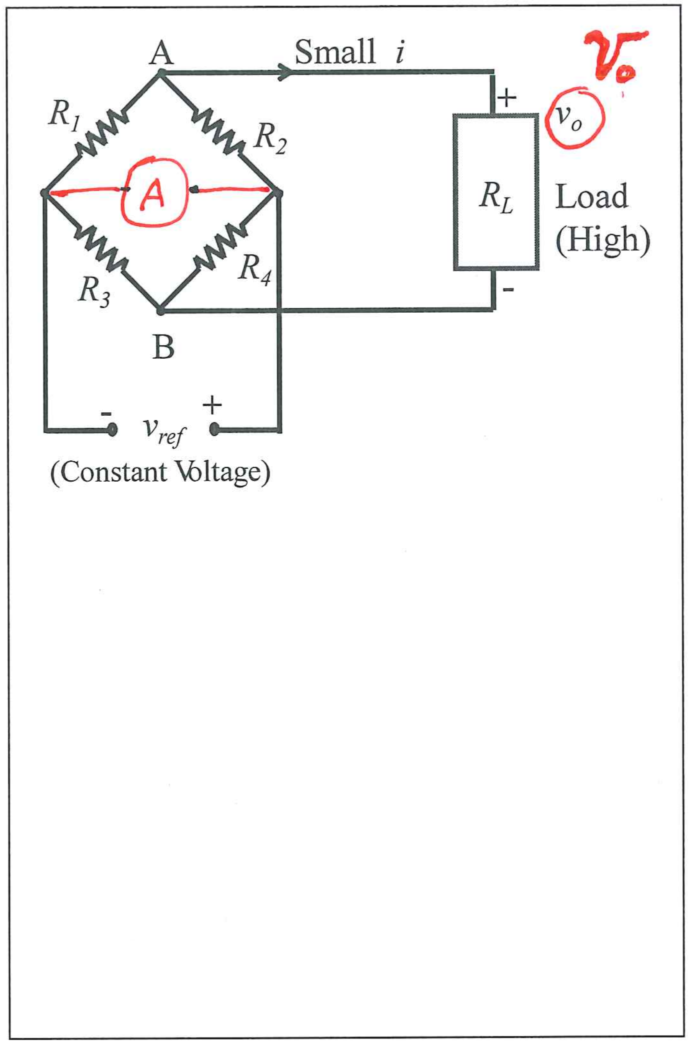
$$V_o = V^*\left[1 - e^{-1/RC\,(t-t^*)}\right] = \frac{1}{2} \Rightarrow e^{-\frac{1}{RC}(t-t^*)}$$

$$\therefore\ t - t^* = -RC\ln\left(\tfrac{1}{2}\right) = 0 = 0.637.\ (RC)$$

If RC gets Larger System Responds slower
RC " smaller " " faster.



Bridge Circuits:



Bridge circuits are used to make a form of measurement:

- Change in resistance
- Change in inductance
- Change in capacitance
- Oscillating frequency

D.C. Bridge - All Resistors

A.C. Bridge - Impedances

* Bridge is Balanced iff

(A) show No current flow

* when balanced

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad ; \text{ For Any } R_L$$

Look @ $V_o = V_A - V_B$

$$= \underbrace{\frac{R_1}{R_1+R_2} V_{ref}}_{V_A} - \underbrace{\frac{R_3}{R_3+R_4} V_{ref}}_{V_B}$$

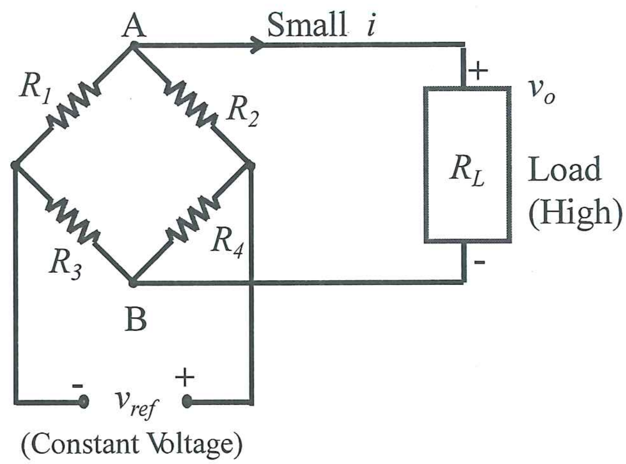
$$= V_{Ref} \left(\frac{\cancel{R_1 R_3} + R_1 R_4 - \cancel{R_3 R_1} - R_3 R_2}{(R_1+R_2)(R_3+R_4)} \right) = \frac{R_1 R_4 - R_3 R_2}{(R_1+R_2)(R_3+R_4)} V_{Ref}$$

Assume $R_1 = R_2 = R_3 = R_4 = R$. $\therefore$ Bridge is balanced. $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Let's increase R_1 by δR, all others @ R

$R \Rightarrow$ The only Active strain gage.

Continuiing.



$$\delta V_o = \frac{[(R+\delta R)R - R^2]}{(R+\delta R+R)(R+R)}\, v_{ref} - 0 \qquad (1)$$

when Balanced

$$\frac{\delta v_o}{v_{Ref}} = \frac{\cancel{R^2} + R\delta R - \cancel{R^2}}{4R^2 + 2R\delta R}$$

$$= \frac{R\delta R}{4R^2 + 2R\delta R}$$

$$= \frac{\delta R}{4R + 2\delta R}$$

$$= \frac{\delta R/R}{4 + 2\frac{\delta R}{R}} \; ; \; \frac{\delta R}{R} : \text{Non Linear}$$

If we assume $\frac{\delta R}{R}$ is Very Small compared to 2; we get Linearized relation.

$$\frac{\delta V_o}{V_{Ref}} = \frac{\delta R}{4R} \; ; \; \frac{1}{4} \Rightarrow \text{Bridge Sensitivity}$$

~25% change

[Active Resistance $R \to R+\delta R$]

$\Rightarrow$ Bridge sensitivity is given by $\frac{\delta V_o}{\delta R}$

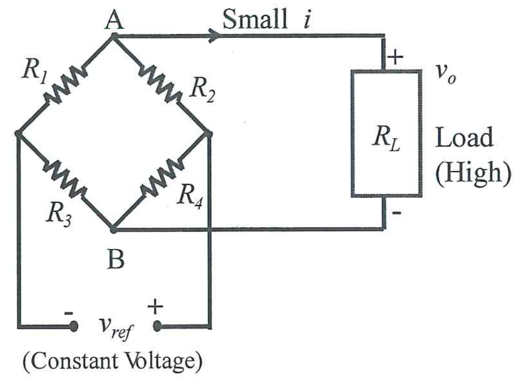
$$\therefore \frac{\delta V_o}{\delta R} = \frac{V_{ref}}{4R} \qquad (2)$$

$$N_p \text{ (Non-Linearity \%ge)} = 100\left(1 - \frac{\text{Linearized output}}{\text{Actual output}}\right)\%$$

From EQ. (1) + (2) $N_p = 50\left(\frac{\delta R}{R}\right)\%$.

Example:

Suppose that in Figure on the right, at first $R_1 = R_2 = R_3 = R_4$. Now increase R_1 by δR, decrease R_2 by δR. This will represent two active elements that act in reverse, as in the case of two strain gage elements mounted on the top and the bottom surfaces of a beam in bending. Show that the bridge output is linear in δR in this case.



Solution:

$R_1 = R + \delta R$, $R_2 = R - \delta R$

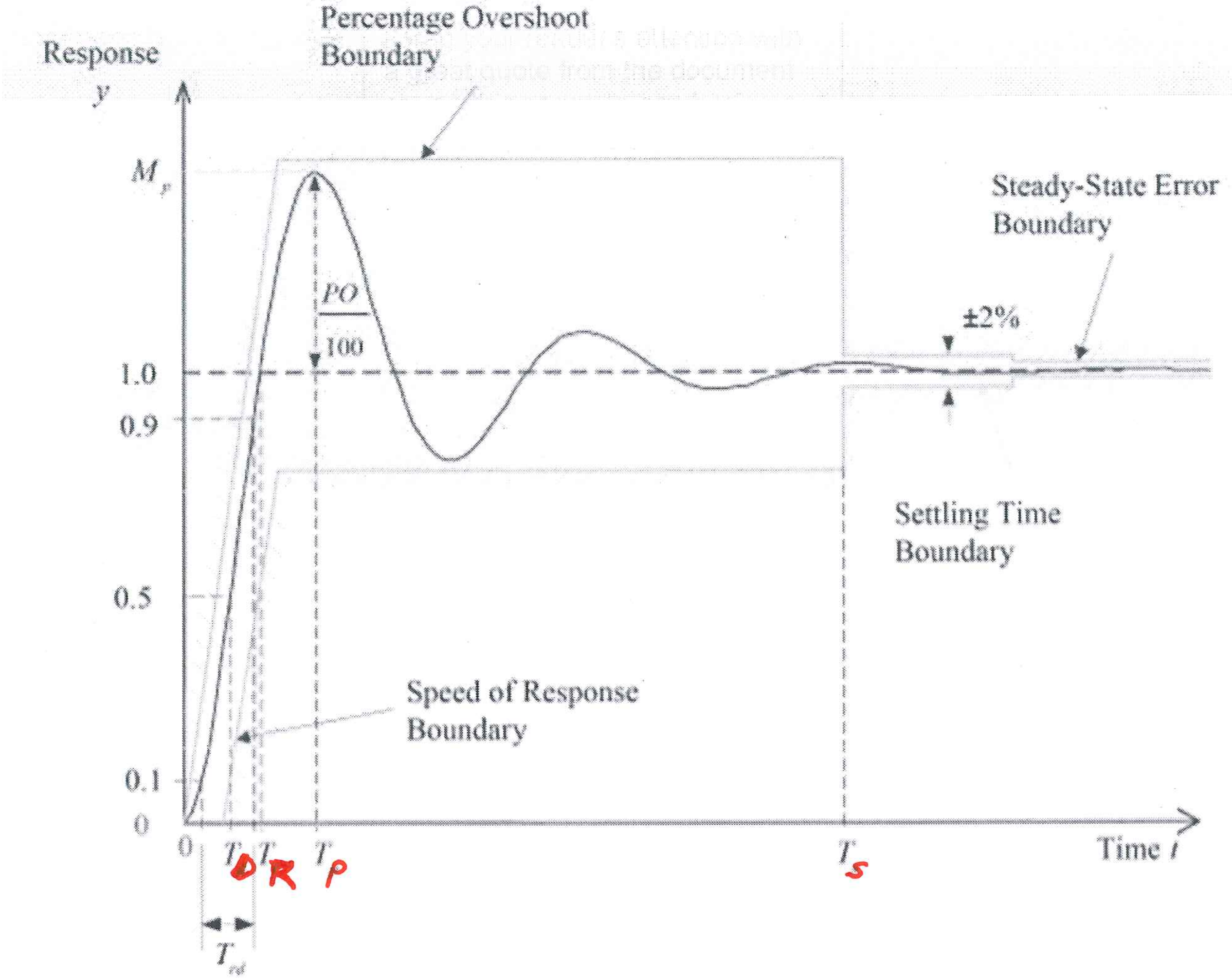
$$v_o = V_A - V_B = \frac{R+\delta R}{(R+\delta R)+(R+\delta R)} V_{ref} - \frac{R}{R+R} V_{ref} .$$

$$\delta v_o = \frac{(R+\delta R)\,2R - R(R+\delta R)(R-\delta R)}{(2R)(2R)} V_{ref}.$$

$$\propto v_o = \frac{2R^2 + 2\delta R - R(R^2 - (\delta R)^2)}{4R^2} V_{ref}$$

$$=$$

$$\frac{\delta v_o}{V_{ref}} = \frac{\delta R}{2R} \quad ; \quad \cdot 0.5\%$$



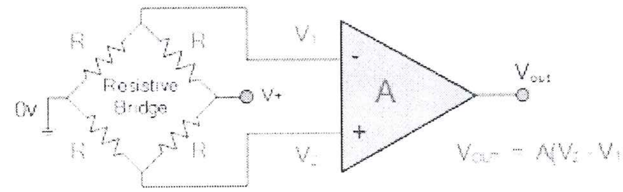
Rise Time $[T_R]$: Time needed to pass 90% of steady state value, first time

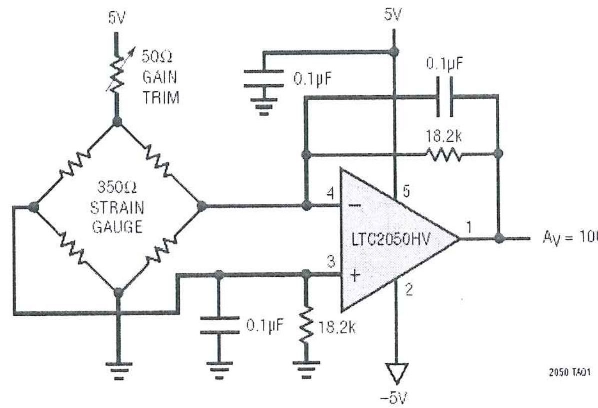
Delay Time $[T_D]$: Time needed to pass 50% of " " .

Peak Time $[T_p]$: Time for first Peak.

Setteling Time $[T_s]$: Time that the signal reaches ±20% of steady state value and remains there.

Bridge Amplifiers:





- The output signal from a resistance bridge is usually very small in comparison to the reference signal, and it has to be amplified to increase its voltage level to a useful value (e.g., for use in system monitoring, data logging, or control).

- This is typically an instrumentation amplifier, which is essentially a sophisticated differential amplifier.

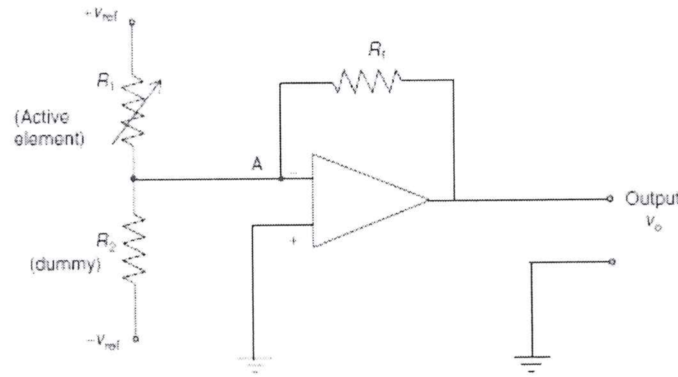
- The bridge amplifier is modeled as a simple gain K_a, which multiplies the bridge output.

Half-Bridge Circuits:

- A half bridge has only two arms.
- Output is tapped from the mid-point of these two arms.
- The ends of the two arms are excited by two voltages, one of which is positive and the other negative.
- Initially, the two arms have equal resistances so that nominally the bridge output is zero.
- One of the arms has the active element. Its change in resistance results in a nonzero output voltage.
- It is noted that the half-bridge circuit is somewhat similar to a potentiometer circuit (a voltage divider).

The two bridge arms have resistances R_1 and R_2, and the output amplifier uses a feedback resistanceR_f.

To get the output equation, we use the two basic facts for an unsaturated op-amp; the voltages at the two input leads are equal (due to high gain), and the current in either lead is zero (due to high input impedance). Hence, voltage at node A is zero and the current balance equation at node A is given by:



$$\frac{V_{ref}}{R_1} + \frac{-V_{ref}}{R_2} + \frac{V_o}{R_f} = 0$$

which means: $V_o = R_f\left(\frac{1}{R_2} - \frac{1}{R_1}\right) V_{ref}$ — (1)

Assume Initially $R_1 = R_2 = R$; Change R_1 by δR

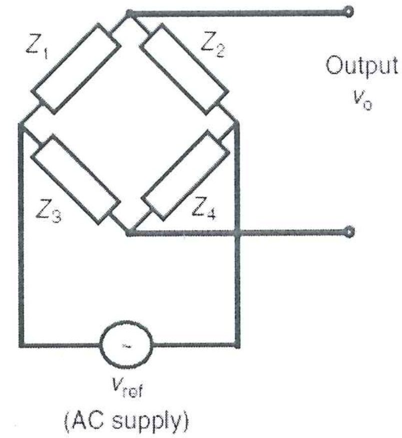
$$\delta V_o = R_f\left(\frac{1}{R} - \frac{1}{R+\delta R}\right) V_{ref} - 0$$ if $R_1 = R_2$ EQ 1=0.

$$\frac{\delta V_o}{V_{ref}} = \frac{R_f}{R}\left(\frac{\cancel{R}+\delta R - \cancel{R}}{R+\delta R}\right) = \frac{R_f}{R}\left(\frac{\delta R/R}{1+\delta R/R}\right)$$

R_f/R is the Amplifier gain ; $N_p = 100 \frac{\delta R}{R} \%$.

⇒ Non-Linearity of half bridge is worse than that of wheatstone bridge

Impedance Bridges:



(a)

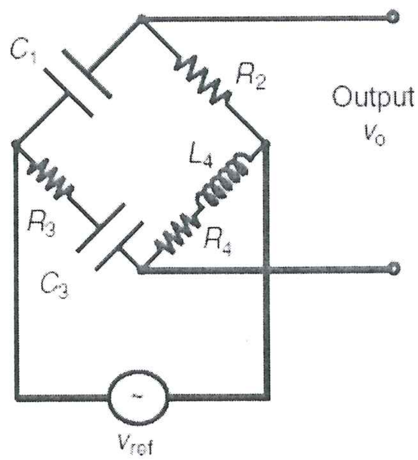
- AC Bridge
- Contains four impedances: Z_1, Z_2, Z_3 *and* Z_4

- V_{ref} : Carrier : AC Supply
- V_o : demodulate
- Bridge can be used to measure X_C, X_L in sensors ∴ Oscillator Circuits as frequency generators

$$v_o(\omega) = \frac{Z_1 Z_4 - Z_2 Z_3}{(Z_1+Z_2)(Z_3+Z_4)} V_{ref}(\omega).$$ AC frequency dependancy

Balanced Condition ⇒ $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$.

Owen Bridge:



Ex. of Measuring C_3 and/oR L_4 by balance Method.

Ind: $v = L\frac{di}{dt}$; Cap: $i = C\frac{dv}{dt}$

in Laplace : V–I transfer function.

Inductor: $\frac{v(s)}{i(s)} = L(s)$; Impedance $Z_L = j\omega L$

Capacitor: $\frac{v(s)}{i(s)} = \frac{1}{C(s)}$; Impedance $Z_C = \frac{1}{j\omega C}$

∴ $Z_1 = \frac{1}{j\omega C_1}$; $Z_2 = R_2$; $Z_3 = R_3 + \frac{1}{j\omega C_3}$; $Z_4 = R_4 + j\omega L_4$; ω = Excitation frequency

For Balanced Bridge $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$; $\cancel{\frac{R_4 + j\omega L_4}{j\omega C_1}}$ or $Z_1 \cdot Z_4 = Z_2 \cdot Z_3$

∴ $\frac{R_4 + j\omega L_4}{j\omega C_1} = R_2\left(R_3 + \frac{1}{j\omega C_3}\right)$; Now Equating Real & Imag. Parts.

$$\left(\frac{R_4}{j\omega C_1} + \frac{\cancel{j\omega} L_4}{\cancel{j\omega} C_1} = R_2 R_3 + \frac{R_2}{j\omega C_3}\right) \Rightarrow \frac{L_4}{C_1} = R_2 R_3 \text{ and } \frac{R_4}{C_1} = \frac{R_2}{C_3}$$

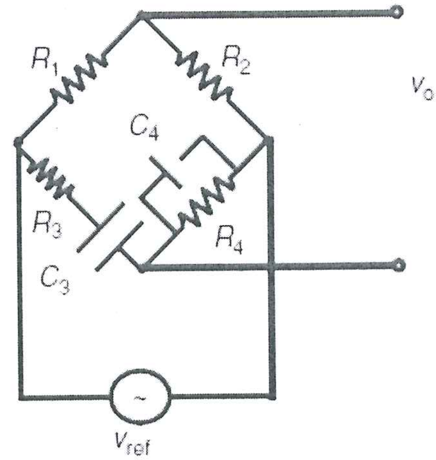
$L_4 = C_1 R_2 R_3$ and $C_3 = \frac{R_2 C_1}{R_4}$

- L_4, C_3 can be obtained by C_1, R_2, R_3, R_4 under balanced conditions.
- Fix C_1 and R_2 : Adjustable R_3 can be used to measure Variable L_4

- Adjustable R_4 can be used to measure Variable C_3

Wien Bridge Oscillator:

In this circuit, we have



$Z_1 = R_1$; $Z_2 = R_2$

$$Z_3 = R_3 + \frac{1}{j\omega C_3}$$

$\frac{1}{Z_4} = \frac{1}{R_4} + j\omega C_4$; Admittance

Bridge to be balanced $\Rightarrow$

$$\frac{R_1}{R_2} = \left(R_3 + \frac{1}{j\omega C_3}\right) \cdot \left(\frac{1}{R_4} + j\omega C_4\right)$$

Equating Real Parts:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} + \frac{C_4}{C_3}$$

Imaginary Parts

$$0 = \omega C_4 R_3 - \frac{1}{\omega C_3 R_4} \quad \left(\text{Note } j = -\frac{1}{j}\right)$$

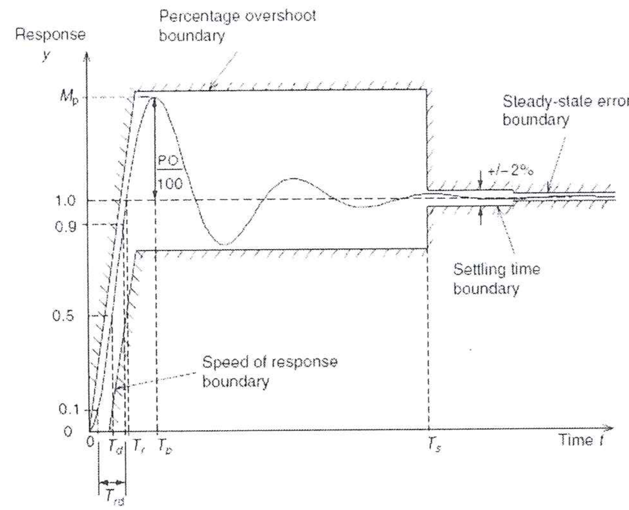
$$\therefore \omega C_4 R_3 = \frac{1}{\omega C_3 R_4} \quad \text{or} \quad \omega^2 = \frac{1}{C_3 R_4 C_4 R_3}$$

or $\omega = \frac{1}{\sqrt{C_3 R_4 C_4 R_3}}$; Oscillator Natural frequency Underbalanced Equations.

- Circuit can be used to measure ~~M~~ unknown Resistance (in strain gage) by measuring (1st) frequency of bridge at Resonance.

Response parameters for time-domain specification of performance:

Delay Time:
This is usually defined as the time taken to reach 50% of the steady-state value for the first time. This parameter is also a measure of speed of response.



Peak Time
The time at the first peak of the device response is the peak time. This parameter also represents the speed of response of the device.

Settling Time
This is the time taken for the device response to settle down within a certain percentage (typically+2%) of the steady-state value. This parameter is related to the degree of damping present in the device as well as the degree of stability.

Percentage Overshoot
This is defined as, $PO = 100(M_P - 1)\%$, using the normalized-to-unity step response curve, where M_P is the peak value. Percentage overshoot (PO) is a measure of damping or relative stability in the device.

Simple Oscillator Model:

- Represents Performance of Various devices
- Depending on the level of damping: Oscillatory and Non-Osc. behaviour can be represented. Model can be expressed as:

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = \omega_n^2 u(t);$$

u: excitation.
ω_n: Undamped Natural f
ξ: Damping Ratio

Damped Natural frq: $\omega_d = \sqrt{1-\xi^2}\,\omega_n$

If excitation is STEP FUNCTION with ZERO initial Conditions is:

$$y = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_d t + \phi)$$

$$\cos\phi = \xi$$

For a transducer: desired to have Small rise time, Small settling time

TABLE 3.1

Time-Domain Performance Parameters Using the Simple Oscillator Model

Performance Parameter	Expression
Rise Time	$T_r = \dfrac{\pi - \phi}{\omega_d}$ with $\cos\phi = \zeta$
Peak Time	$T_p = \dfrac{\pi}{\omega_d}$
Peak Value	$M_p = 1 - e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
Percentage Overshoot (PO)	$PO = 100\, e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
Time Constant	$\tau = \dfrac{1}{\zeta\omega_n}$
Settling Time (2%)	$T_s = -\dfrac{\ln\left[0.02\sqrt{1-\zeta^2}\right]}{\zeta\omega_n} \approx 4\tau = \dfrac{4}{\zeta\omega_n}$

An automobile weighs 1000 kg. The equivalent stiffness at each wheel, including the suspension system, is approximately 60.0×10^3 N/m. If the suspension is designed for a percentage overshoot of 1%, estimate the damping constant that is needed at each wheel.

Solution: using a simple oscillator model of the form

each W: $1000/4 = 250$

$m\ddot{y} + b\dot{y} + ky = ku(t)$; m = mass, = 250, b = damping constant

k = equivalent stiffness = 60×10^3 N/m

u = displacement excitation @ each wheel

P.O = 1%.

$250\ddot{y} + b\dot{y} + ky = ku(t)$

$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \left(\frac{k}{m}\right)u(t)$

$\frac{k}{m} = \omega_n^2 \rightarrow \omega_n = \sqrt{\frac{k}{m}}$; $\frac{b}{m} = 2\zeta\omega_n \Rightarrow \zeta = \dfrac{b}{2\sqrt{km}}$

From Table above (for percent overshoot - P.O)

$1 = 100\, e^{\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \Rightarrow \zeta = 0.83$, substitute

$0.83 = \dfrac{b}{2\sqrt{60\times10^3 \times 250}} = 6.43\times10^3$ N/m/sec.

Active Transducer:

- External power is required to operate active sensors/transducers, and they do not depend on their own power conversion characteristics for operation.
- A good example for an active device is a resistive transducer, such as a potentiometer, which depends on its power dissipation through a resistor to generate the output signal.
- Note that an active transducer requires a separate power source (power supply) for operation,

Passive transducer:

- Draws its power from a measured signal (measurand).
- Since passive transducers derive their energy almost entirely from the measurand, they generally tend to distort (or load) the measured signal to a greater extent than an active transducer would. Precautions can be taken to reduce such loading effects.
- On the other hand, passive transducers are generally simple in design, more reliable, and less costly.
- For example, a piezoelectric charge generation is a passive process. But, a charge amplifier, which uses an auxiliary power source, would be needed by a piezoelectric device in order to condition the generated charge.

Error Analysis:

- Error = (instrument reading) – (true value)
- Measurement Accuracy: Determines the closeness of the measured value to the true value
- Instrument Accuracy: Related to the worst accuracy obtainable within the dynamic range of the instrument in a specific operating environment

More discussion on Active and Passive Sensors:

- ***An active sensor*** is a sensing device that requires an external source of power to operate; active sensors contrast with passive sensors, which simply detect and respond to some type of input from the physical environment.

- In the context of remote sensing, an active sensor is a device with a transmitter that sends out a signal, light wavelength or electrons to be bounced off a target, with data gathered by the sensor upon their reflection.

- Active sensors are also widely used in manufacturing and networking environments for example to monitor industrial machines or data center infrastructure so anomalies can be detected and components can be repaired or replaced before they break and shut everything down.

- ***Examples of other active sensor-based technologies include***: scanning electron microscopes, radar, GPS, x-ray, sonar, infrared and seismic. However, as can be the case with some sensors, seismic and infrared light sensors exist in both active and passive forms.

- ***A passive sensor*** is a device that detects and responds to some type of input from the physical environment.

- Passive sensor technologies gather target data through the detection of vibrations, light, radiation, heat or other phenomena occurring in the subject's environment.

- They contrast with active sensors, which include transmitters that send out a signal, a light wavelength or electrons to be bounced off the target, with data gathered by the sensor upon their reflection.

- Sensors can also be used in harsh environments and places inaccessible to people.

- ***Examples of passive sensor-based technologies include***: Photographic, thermal, electric field sensing, chemical, infrared and seismic. However, as can be the case with some sensors, seismic and infrared light sensors exist in both active and passive forms.

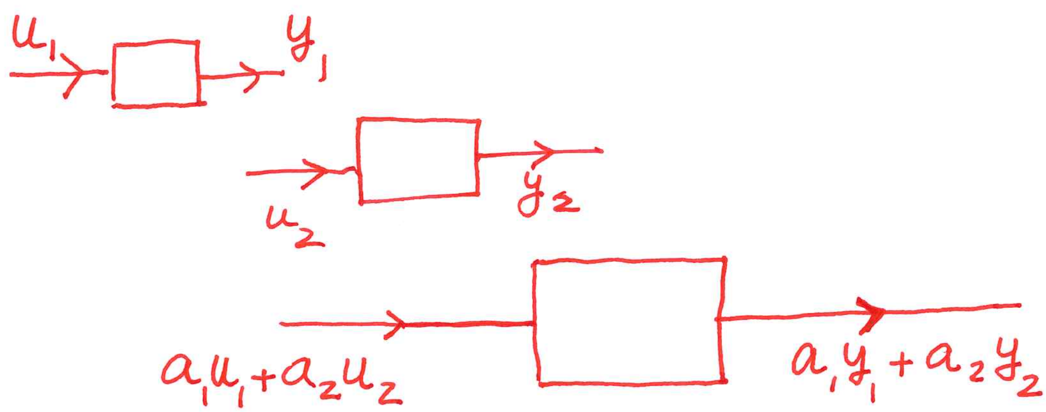
Linearizing Devices:

- Nonlinearity is present in any physical device, to varying levels.

- If the level of nonlinearity in a system (component, device, or equipment) can be neglected without exceeding the error tolerance, then the system can be assumed linear.

- Linear system is one that can be expressed as one or more linear differential equations.

- Note that the principle of superposition holds for linear systems.



Nonlinearities in a system can appear in two forms:

- 1. Dynamic manifestation of nonlinearities
- 2. Static manifestation of nonlinearities

Cases:

- The useful operating region of a system can exceed the frequency range where the frequency response function is flat. The operating response of such a system is said to be dynamic.
    - Examples include a typical control system (e.g., automobile, aircraft, milling machine, robot), actuator (e.g., hydraulic motor), and controller (e.g., proportional-integral-derivative or PID control circuitry).

- Nonlinearities of such systems can manifest themselves in a dynamic form such as the jump phenomenon (also known as the fold catastrophe), limit cycles, and frequency creation.

Solutions for dynamic manifestations of nonlinearity:

- Design changes, extensive adjustments, or reduction of the operating signal levels and bandwidths would be necessary in general, to reduce or eliminate.

Is that a good Solution?

- In many instances, such changes would not be practical, and we may have to *somehow cope with the presence of these nonlinearities* under dynamic conditions.

- Design changes might involve:

    - Replacing conventional gear drives by devices such as harmonic drives to reduce backlash.

    - Replacing nonlinear actuators by linear actuators, and

    - Using components that have negligible Coulomb friction and that make small motion excursions.

What is Coulomb Friction?

- Coulomb friction is a simplified quantification of the friction force that exists between two dry surfaces in contact with each other.

- All friction calculations are approximations, and this measurement is dependent only on the fundamental principles of motion.

- It assumes that the contact surfaces are fairly uniform and that the coefficient of friction that must be overcome for motion to begin is well-established for the materials in contact.

What about Static Manifestations:

- Making design changes and adjustments, as in the case of dynamic devices.

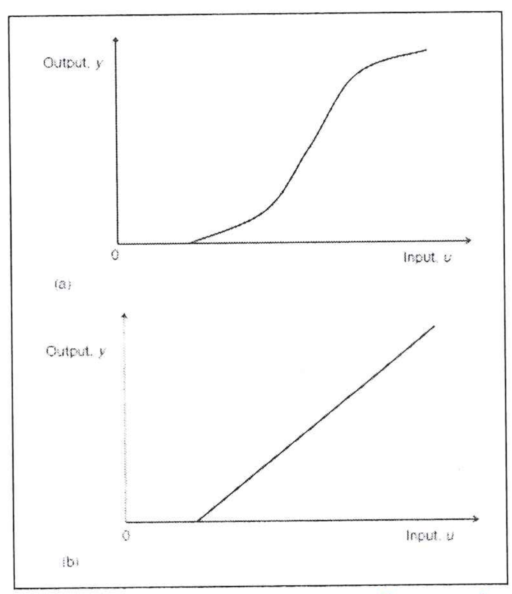
- Since the response is static, and since we normally deal with an available device (fixed design) whose internal hardware cannot be modified,

- We should consider ways of linearizing the input/output characteristic by ***modifying the output*** itself.
  - Linearization using digital software
  - Linearization using digital (logic) hardware
  - Linearization using analog circuitry

- **In the software approach to linearization:**
  - Output of the device is read into a digital processor with software-programmable memory
  - ***And the output is modified*** according to the program instructions.

- **In the hardware approach:**
  - Output is read by a device with ***fixed logic circuitry for processing (modifying)*** the data.

- **In the analog approach:**
  - A ***linearizing circuit is directly connected at the output of the device***, so that the ***output of the linearizing circuit is proportional to the input*** to the original device.



Simple Curve: Add DC

DC component will convert the characteristics into Linear form given by: $y = ku$

This method is called offsetting: Can't use for More Complex Curves.

Software based linearization:

Assuming that the nonlinear relationship between the input and the output of a nonlinear device is known, the input can be computed for a known value of the output.

In the software approach of linearization, a processor and memory that can be programmed using software (i.e., a digital computer) is used to compute the input using output data.

non-Linear charactris given by

$$y = f(u) : \begin{array}{l} u \rightarrow \text{device input} \\ y \rightarrow \text{device output} \end{array}$$

Assuming one-one Relation, Inverse Equation.

$$u = f^{-1}(u)$$

Analysis:

- Flexible - Linearization algorithm can be modified (e.g., improved, changed) simply by modifying the program stored in the RAM.
- Highly complex nonlinearities can be handled by the software method.
- Relatively slow.

Linearization by Hardware Logic:

- Hardware logic method:
  - Linearization algorithm is permanently implemented in the IC form using appropriate digital logic circuitry for data processing and memory elements (e.g., flip-flops).

- However, ***algorithm and numerical values of parameters*** (except input values) cannot be modified without redesigning the IC chip, because a hardware device typically does not have programmable memory.

- Difficult to implement very complex linearization algorithms –Mass chip production, initial chip development cost? Testing for our needs only?

- Lack of Flexibility - A digital linearizing unit with a processor and a read-only memory (ROM), whose program cannot be modified, also lacks the flexibility of a programmable software device.

Table Look up: Fast, Accurace $\propto$ Amount of stored data
Memory Intensive; Increased Accuracy Vs. Speed & Memory Requirements.

Analog Linearizing Circuitry

Three types of analog linearizing circuitry can be identified:

- Offsetting circuitry
- Circuitry that provides a proportional output
- Curve shapers

Offsetting circuitry:

- An offset is a nonlinearity that can be easily removed using an analog device.
- Adding a dc offset of equal value to the response, in the opposite direction. Deliberate addition of an offset in this manner is known as offsetting.
- The associated removal of original offset is known as offset compensation.
- Example:
  - Results of ADC and DAC can be removed by analog offsetting.
  - Constant (dc) error components, such as steady-state errors in dynamic systems due to load changes, gain changes, and other disturbances, can be eliminated by offsetting.

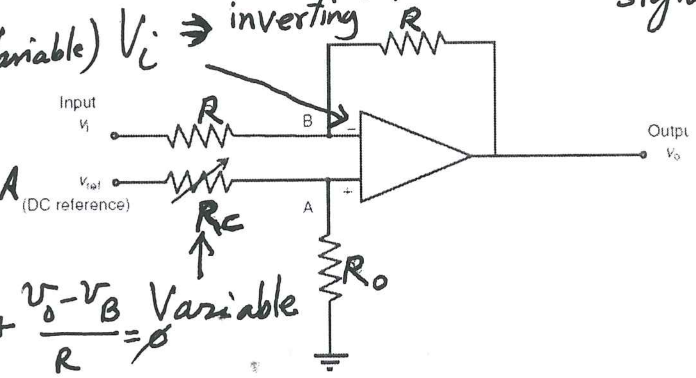
Easiest Approach - Use Summer Op-Amp (Add or subtract)

input will appear in V_o with reversed sign.

V_{ref} : Provides offsetting Voltage (Variable) $V_i \Rightarrow$ inverting

R_c : Compensator (Variable)

Current balancing Equation: Node-A



Node-A

$$\frac{V_{ref} - V_A}{R_C} = \frac{V_A}{R_o}$$

① $$V_A = \left(\frac{R_o}{R_o + R_c}\right) v_{ref}$$

Node-B

$$\frac{v_i - v_B}{R} + \frac{v_o - v_B}{R} = 0$$

OR

$$v_o = -v_i + 2v_B \quad \text{②}$$

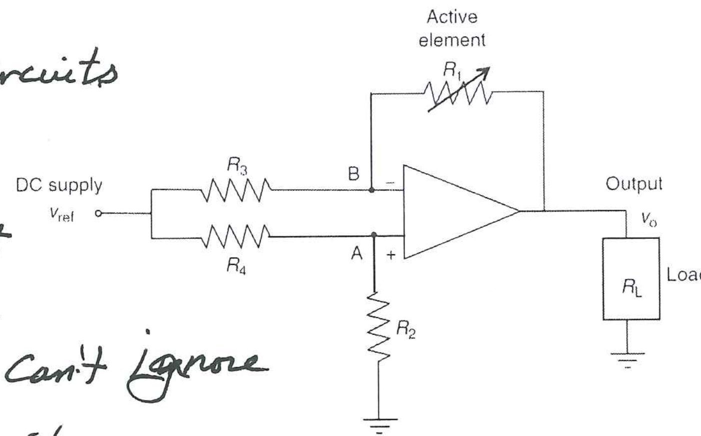
Since $V_A = V_B$ for op-Amp we can substitute ① into ②

$$\therefore \quad V_o = -V_i + \frac{2R_o}{(R_o + R_c)} V_{ref}$$

Can be reversed @ input or connecting to other circuitry and recovering

offsetting term by chosing R_c.

Proportional-Output Circuitry:



- Recall $\dfrac{\delta v_o}{V_{ref}}$, $\dfrac{\delta R}{R}$ in bridge Circuits
- If $\delta R << R$: Non linear Circuits can be linearized without introducing large errors.
- If δR is not Small Vs R: Can't ignore

<u>Solution</u>: Linearize the circuit.

Node-A

$$\frac{V_{ref}-V_A}{R4} = \frac{V_A}{R_2}$$

$$\frac{V_{ref}}{R4} - \frac{V_A}{R4} = \frac{V_A}{R_2}$$

$$V_A = \left(\frac{R_2}{R_2+R_4}\right) V_{ref}$$

Node-B

$$\frac{V_{ref}-V_B}{R_3} + \frac{V_o - V_B}{R_1} = 0$$

$$\frac{V_{ref}}{R_3} - \frac{V_B}{R_3} + \frac{V_o}{R_1} - \frac{V_B}{R_1} = 0$$

$$V_B = \frac{R_1 V_{ref} + R_3 V_o}{R_1+R_3}$$

Since $V_A = V_B$.

$$\frac{R_1 V_{ref} + R_3 V_o}{(R_1+R_3)} = \frac{R_2}{(R_2+R_4)} V_{ref} \text{ ; Cross Multiplying}$$

$$\frac{R_2(R_1+R_3)}{R_2+R_4} V_{ref} = R_1 V_{ref} + R_3 V_o$$

$$R_3 V_o = \frac{R_2(R_1+R_3)}{R_2+R_4} V_{ref} - R_1 V_{ref}$$

$$R_3 V_o = \frac{(\cancel{R_2 R_1} + R_2 R_3 - \cancel{R_1 R_2} - R_1 R_4)}{R_1+R_4} V_{ref}$$

$$\therefore V_o = \left(\frac{R_2 R_3 - R_1 R_4}{R_1+R_4}\right) V_{ref}$$

; Similar to wheatstone bridge output.

Wheatstone
Equation $v_o = v_A - v_B = \frac{R_1 v_{ref}}{(R_1+R_2)} - \frac{R_3 v_{ref}}{(R_3+R_4)} = \frac{(R_1R_4 - R_2R_3)}{(R_1+R_2)(R_3+R_4)} v_{ref}.$
2.88 (text)

$$V_0 = \frac{R_2R_3 - R_1R_4}{R_3(R_2+R_4)} v_{ref}$$

$V_0 = 0$ if $R_1 = R_2 = R_3 = R_4 = R$ (in beginning).

∴ Circuit is balanced

Let's change R_1 by δR

$$\delta V_0 = \frac{R^2 - (R+\delta R)R}{R(R+R)} v_{ref} - 0$$

$$\text{or } \delta V_0 = \frac{\cancel{R^2} - \cancel{R^2} - R\delta R}{2R^2} v_{ref}$$

$$\frac{\delta v_0}{v_{ref}} = -\frac{\cancel{R}\delta R}{2R^2} = -\frac{\delta R}{2R}.$$

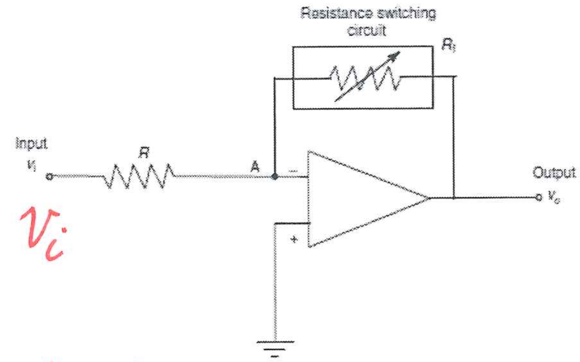
$$\therefore \frac{\delta v_0}{v_{ref}} = -\frac{1}{2}\frac{\delta R}{R}$$

Clearly $\delta v_0 \propto \delta R$

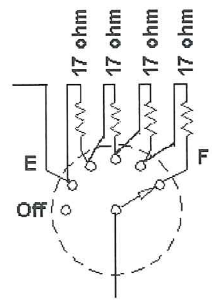
Sensitivity $(\frac{1}{2})$ is twice that of wheatstone $(\frac{1}{4})$
with one active element.

Curve Shaping Circuitry:

- Sort of like an amplifier with adjustable gain.
- Adjustable Feedback resister R_f
- Bank of resistors and automatic switching can be deployed using Zener diodes.



- Amplifier Gain Variable in discrete steps
- One can use a potentiometer as R_f (feed back resistor) so the gain can be adjusted continuous
- Various Resistor steps ⇒ Diff V



Looking at Node-A.

$$\frac{V_i}{R} + \frac{V_o}{R_f} = 0$$

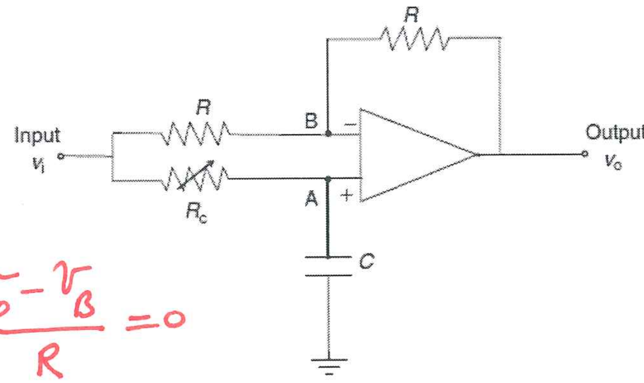
$$V_o = -\frac{R_f}{R} V_i$$

$$\therefore \quad V_o \propto R_f \quad \text{(ignore -ve sign)}$$

Phase Shifters:

phase Angle $\propto R_c$?

Starting with Nodes A & B.



Node A

$$\frac{v_i - V_A}{R_c} = \frac{C\, dv_A}{dt}$$

$$V_i = v_A + R_c C \frac{dv_A}{dt}$$

Laplace !!

$$v_i = V_A + R_c C\, v_A s$$

$$v_i = V_A(1+\tau s) \quad -①$$

Node B

$$\frac{v_i - V_B}{R} + \frac{v_o - v_B}{R} = 0$$

$$\frac{V_i}{R} - \frac{V_B}{R} + \frac{v_o}{R} - \frac{v_B}{R} = 0$$

$$V_B = \frac{1}{2}(v_i + v_o) \quad -②$$

Ideal op Amp $v_A = v_B$; Sub ② into ①

$$V_i = \frac{1}{2}(v_i + v_o)(1+\tau s) \; ; \quad \tau = R_c C \text{ : Time constant.}$$

Transfer function. G(s) can be evaluated as follows:

$$\frac{v_o}{v_i} = G(s) = \frac{1-\tau s}{1+\tau s}$$

$$\text{Magnitude } |G(j\omega)| = \frac{\sqrt{1+\tau^2\omega^2}}{\sqrt{1+\tau^2\omega^2}} = 1.$$

$$\text{phase Angle } \angle G(j\omega) = -\tan^{-1}\tau\omega - \tan^{-1}\tau\omega$$

$$= -2\tan^{-1}\tau\omega$$

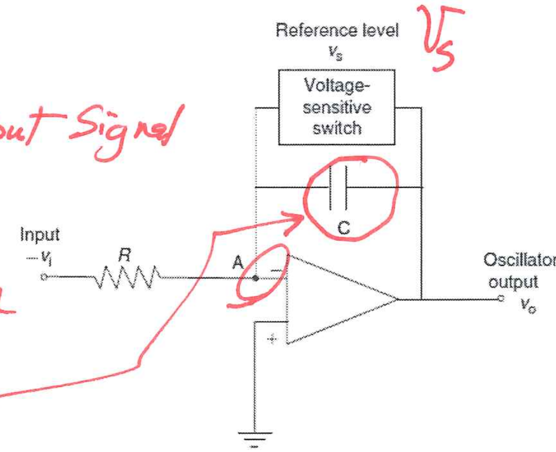
$$= -2\tan^{-1}R_c C\,\omega$$

$\therefore$ Phase Angle $\propto R_c$

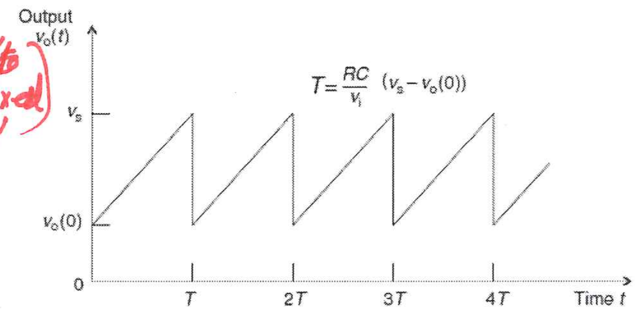
Magnitude = 1 $\Rightarrow$ Undistorted; Angle being −ve $\Rightarrow$ phase lag.

and Phase lag $\propto R_c$

Voltage to Frequency Convertor: (VFC)



- VFC produces a periodic output signal output signal 'f' $\propto$ level of V_i
- Also called VCO: Voltage Controlled Oscillator
- Most Common: use a CAP.
- Time required for CAP to charge to fixed V is inversely proportional to charging Voltage.



Node-A $\quad \frac{v_i}{R} = C \frac{dv_o}{dt}$

$$V_o(t) = \frac{1}{RC} v_i(t) + v_o(0)$$

Switch closes when $v_o(t) = V_s$ (ref); CAP discharges

CAP charging time T is given by

$$V_s = \frac{1}{RC} V_i T + V_o(0)$$

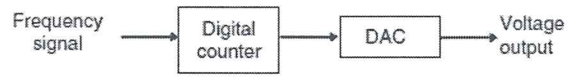
$$\therefore \quad T = \frac{RC}{V_i}\left(V_s - V_o(0)\right)$$

Switch opens again to charge / closes to discharge

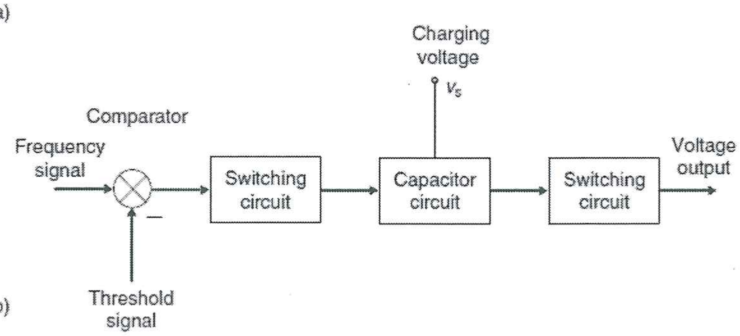
$\Rightarrow$ sawtooth wave

$$f = \frac{V_i}{RC(V_s - V_o(0))}$$

Frequency to Voltage Convertor



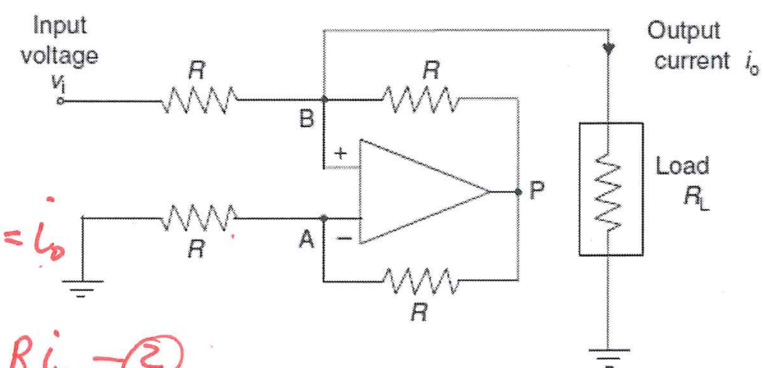
(a)



(b)

* Use digital counter to count signal frequency and use DAC to obtain voltage proportional to frequency
* Frequency signal supplied to comparator with threshold voltage level.
- output depends on which level is higher or lower than other. (sign)
- First sign change (– to +) in comparator will trigger switching and connect to charge CAP.
- Next (+ to –) change will short CAP & discharge.
- level of charge is dependant on switching period.
- used for demodulation

Voltage to Current Convertor:



Node-A

$$\frac{V_A}{R} = \frac{V_P - V_A}{R}$$

$$\Rightarrow 2V_A = V_P \quad (1)$$

Node-B

$$\frac{V_i - V_B}{R} + \frac{V_P - V_B}{R} = i_o$$

$$\Rightarrow V_i - 2V_B + V_P = R\,i_o \quad (2)$$

using fact $V_A = V_B$
sub (1) into (2)

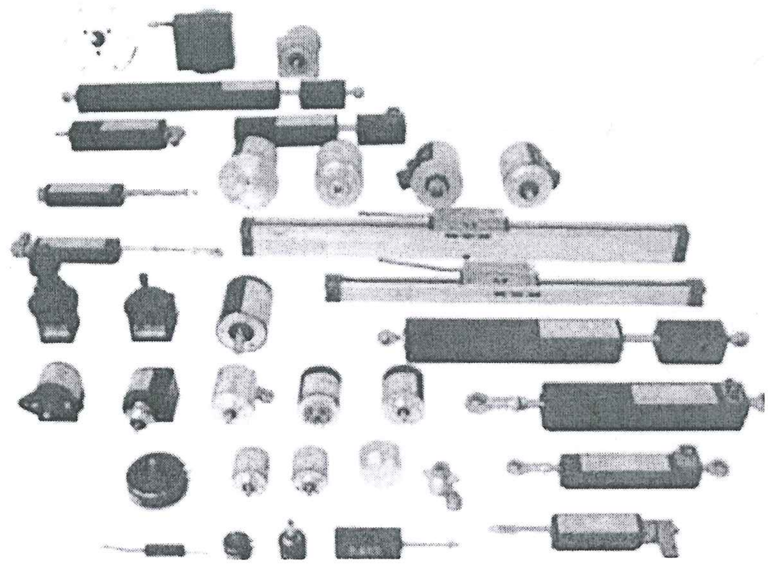
$$i = \frac{V_i}{R} \qquad \therefore\ i \propto V_i$$

Motion Transducers:

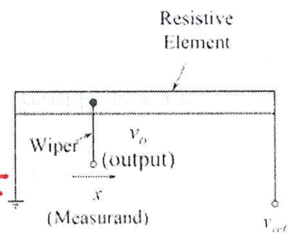
- Potentiometers (resistively coupled)
- Variable inductance (electromagnetically coupled)
- Variable capacitance
- Eddy current
- Piezoelectric

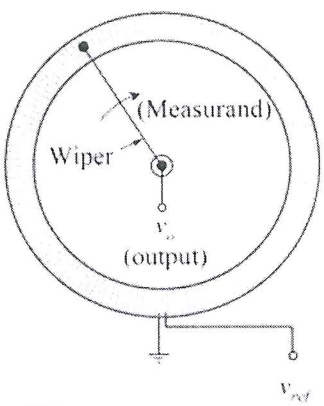
Potentiometer:

- Uniform coil of wire or a film of high resistive material- Carbon, platinum, or conductive plastic.

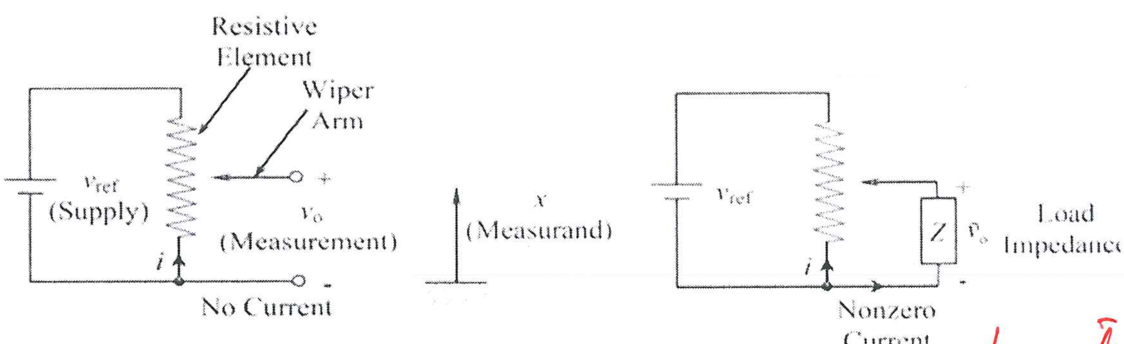


Recti linear





Angular Motions

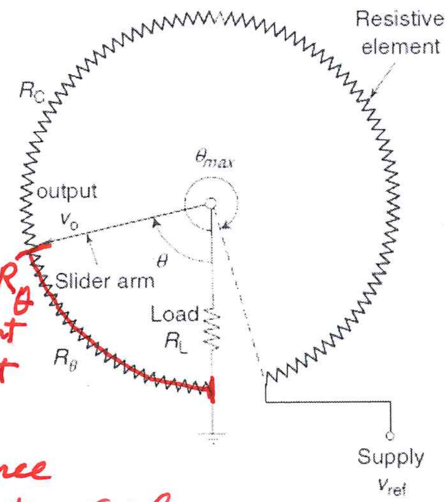


Loading

As load is connected Vref will change?

Loading Nonlinearity:

- What is the significance of the electrical loading nonlinearity error caused by a purely resistive load connected to the pot?



- For general position θ of Arm, let $R = R_\theta$ in Output Segment
- Assuming Coil is Uniform

$$\therefore \; R_\theta = \frac{\theta}{\theta_{max}} R_C \quad (1)$$

$\leftarrow$ Total Resistance of Potentiometer Coil

- Current Balance @ Sliding Node (pt.) is:

$$\frac{V_{ref} - V_o}{R_C - R_\theta} = \frac{V_o}{R_\theta} + \frac{V_o}{R_L}$$

$\leftarrow$ Load Resistor

from here

Multiply by R_C + use (1)

$$\frac{V_{ref} - V_o}{1 - \frac{R_\theta}{R_C}} = \frac{V_o}{\frac{R_\theta}{R_C}} + \frac{V_o}{\frac{R_L}{R_C}}$$

θ / θ_{max}

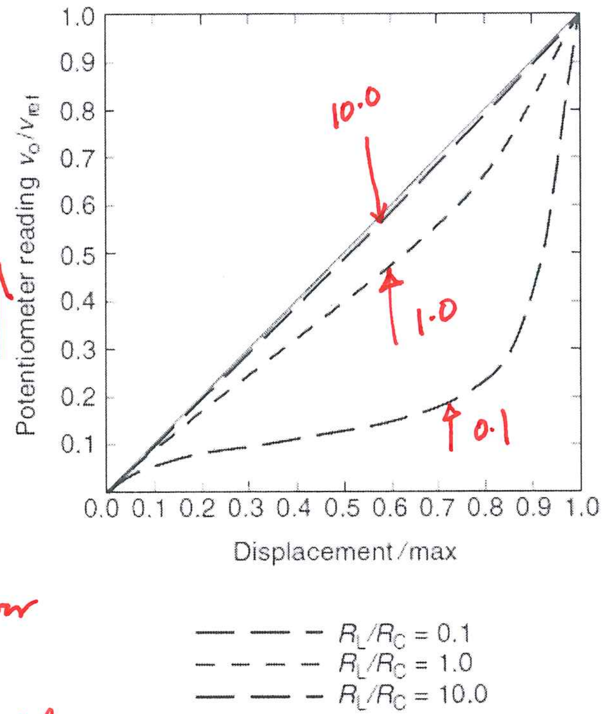
$$\therefore \frac{V_{ref} - V_o}{1 - \frac{\theta}{\theta_{max}}} = \frac{V_o}{\theta / \theta_{max}} + \frac{V_o}{R_L / R_C}$$

$$\frac{V_{ref}}{\frac{\theta_{max} - \theta}{\theta_{max}}} = \frac{V_o}{\frac{\theta_{max} - \theta}{\theta_{max}}} + \frac{V_o}{\theta / \theta_{max}} + \frac{V_o}{R_L / R_C}$$

Applying Algebra:

$$\frac{V_o}{V_{ref}} = \left[\frac{(\theta / \theta_{max})(R_L / R_C)}{(R_L / R_C + (\theta / \theta_{max}) - (\theta / \theta_{max})^2)} \right]$$

$$\frac{V_o}{V_{ref}} = \left[\frac{\left(\theta/\theta_{max}\right)\left(R_L/R_C\right)}{\left(\frac{R_L}{R_C} + \left(\theta/\theta_{max}\right) - \left(\theta/\theta_{max}\right)^2\right)}\right]$$



- High Loading Error for Low $\frac{R_L}{R_C}$ ratio.
- Good Accuracy for $R_L/R_C > 10$ & Small Values of θ/θ_{max}

What to do to reduce loading Error in Pots?

- Increase R_L/R_C: Load Impedance V_s Coil Impedance.
- Use Pots to measure Small values of θ/θ_{max}; Calibrate Only a Small Segment of Resistance element for Linear reading

Loading-non-linearity is given by

$$e = \frac{\left(\frac{V}{V_{ref}} - \theta/\theta_{max}\right)}{\theta/\theta_{max}} * 100\%$$

$\frac{R_L}{R_C}$	$\underline{\underline{e}}$ @ $\theta/\theta_{max} = 0.5$
0.1	−71.4%
1.0	−20%
10.0	−2.4%

<u>Resolution:</u> defined by # of turns in COIL.
Coil of N turns, resolution $r = \frac{100}{N}\%$

Example:

A high-precision mobile robot uses a potentiometer attached to the drive wheel to record its travel during autonomous navigation. The required resolution for robot motion is 1 mm, and the diameter of the drive wheel of the robot is 20 cm. Examine the design considerations for a standard (single-coil) rotatory potentiometer to be used in this application.

1 mm = 0.1 cm; Diameter = 20 cm ∴ r = 10

Circumference = $2\pi(10) = 20\pi$.

Solution:

- Assuming POT is directly connected to drive wheel.
- Required POT resolution; $\frac{0.1 \text{ cm}}{20\pi} * 100\% = 0.16\%$
- using $r = \frac{100}{N}\%$ $\Rightarrow N = \frac{100}{0.16} = 625$ turns
- Pot Dia = 10 cm; Wire diameter = d
- Potentiometer circumference = $\pi * 10 = 625 * d$

$$\therefore d = 0.5 \text{ mm}$$

- Assume POT resistance = 5Ω

  Wire Resistivity = 4 μΩ cm

  Diameter of Core = $\frac{4 * 10^{-6} \times \pi D * 625}{\pi (0.05/2)^2} = 5\Omega$

- Resistivity = resistance * (X-Sectional Area)/Length

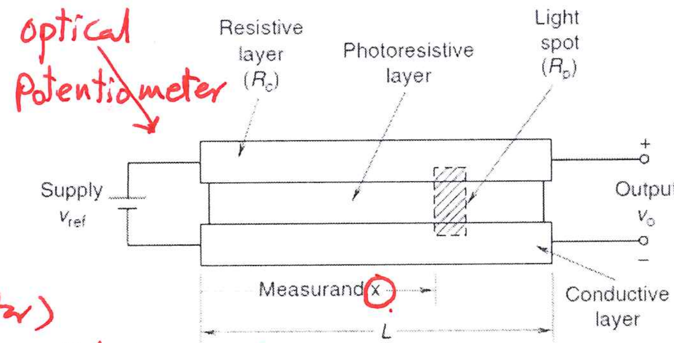
$$\therefore D = 1.25 \text{ cm}$$

- Potentiometer Sensitivity $S = \frac{\Delta v_o}{\Delta \theta}$; $\frac{\text{Change in } v_o}{\text{Change in } \theta}$

Read Example 4.2 (first Edition) Text.

Optical Potentiometer:
The optical potentiometer, shown is a displacement sensor. A layer of photo-resistive material is sandwiched between a layer of ordinary resistive material and a layer of conductive material.

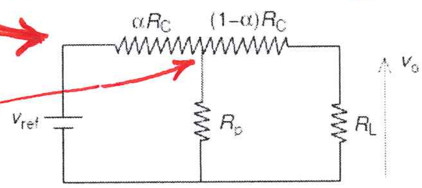
optical Potentiometer



(a)

* At equivalent ckt, R_L (Load Resistor) is present at the output of POT

Equivalent Circuit $\alpha = x/L$



(b)

* Current through Load V_o/R_L
* $\alpha = \frac{x}{L}$; Fractional Position of light spot
* Current balance @ Junction:

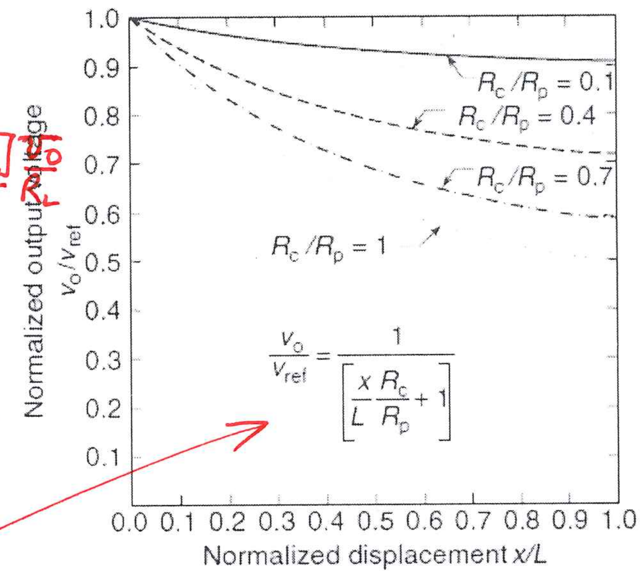
$$\frac{V_{ref} - \left[(1-\alpha)R_c + R_L\right]\frac{V_o}{R_L}}{\alpha R_c} = \frac{V_o}{R_L} + \frac{\left[(1-\alpha)R_c + R_L\right]\frac{V_o}{R_L}}{R_p}$$

Can be re-written as:

$$\frac{V_o}{V_{ref}}\left\{\frac{R_c}{R_L} + 1 + \frac{x}{L}\frac{R_c}{R_p}\left[\left(1-\frac{x}{L}\right)\frac{R_c}{R_L} + 1\right]\right\} = 1$$

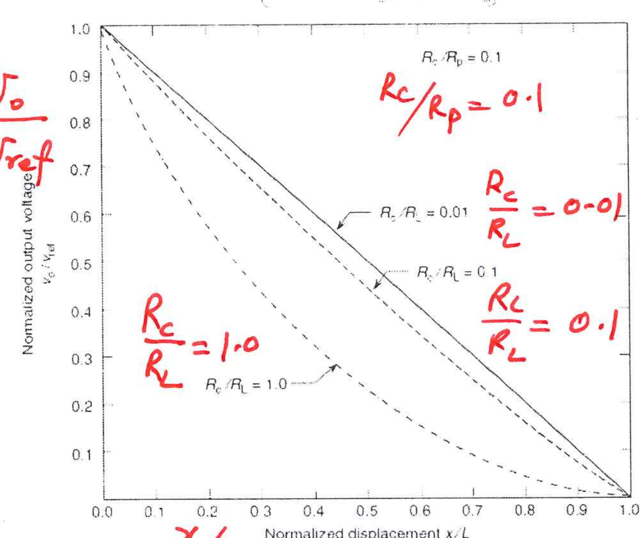
If $R_L \gg R_c$; $R_c/R_L \simeq 0$

$$\therefore \frac{V_o}{V_{ref}} = \frac{1}{\left[\frac{x}{L}\cdot\frac{R_c}{R_p} + 1\right]};$$



$$\frac{v_o}{v_{ref}}\left\{\frac{R_c}{R_L} + 1 + 0.1\frac{x}{L}\left[\left(1-\frac{x}{L}\right)\frac{R_c}{R_L} + 1\right]\right\} = 1$$

→ Original Equation is plotted → $\frac{V_o}{V_{ref}}$

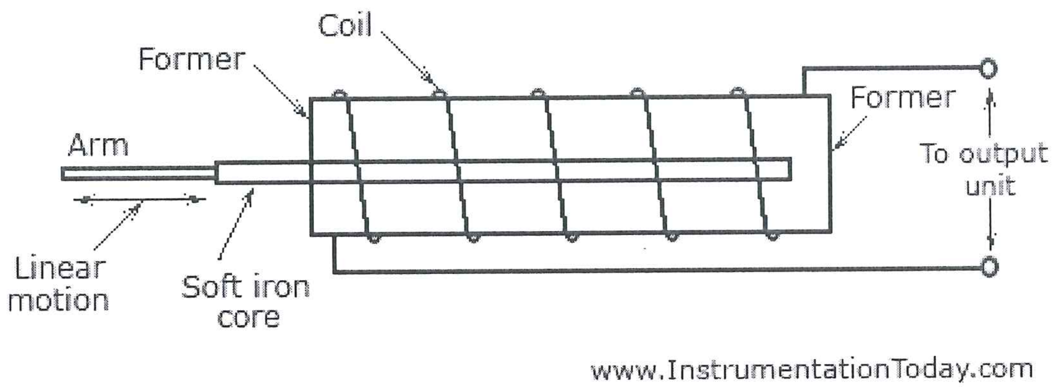


$R_c/R_p = 0.1$; $\frac{R_c}{R_L} = 0.01$; $\frac{R_c}{R_L} = 0.1$; $\frac{R_c}{R_L} = 1.0$; x/L

* Behaviour of optical POT becomes more linear for higher Values of load resistance

Variable Inductance Transducers:

- When the flux linkage (defined as magnetic flux density times the number of turns in the conductor) through an electrical conductor changes, ***a voltage in proportion to the rate of change of flux*** is induced in the conductor.



- This voltage in turn, generates a magnetic field, which opposes the original (primary) field. Hence, a mechanical force is necessary to sustain the change of flux linkage.

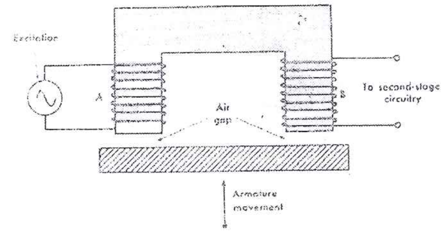
- If the change in flux linkage is brought about by a relative motion, the associated mechanical energy is directly converted (induced) into electrical energy.

- This is the basis of electromagnetic induction, and it is the principle of operation of electrical generators and variable-inductance transducers.

- ***The induced voltage or change in inductance may be used as a measure of the motion.***

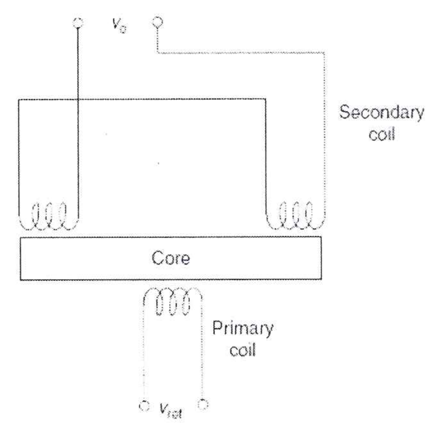
Three primary types can be identified as:
  - Mutual-induction transducers
  - Self-induction transducers
  - Permanent-magnet transducers.

Mutual-induction transducers:

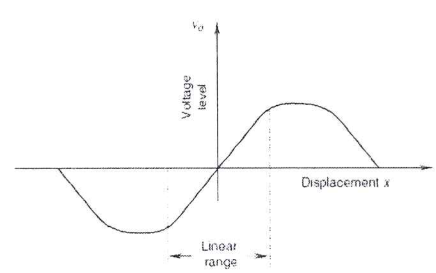


- Arrangement of a mutual-induction transducer constitutes two coils, the primary winding and the secondary winding. One of the coils (primary winding) carries an alternating-current (ac) excitation, which induces a steady ac voltage in the other coil (secondary winding).
- The level (amplitude, rms-value, etc.) of the induced voltage depends on the flux linkage between the coils.
- None of these transducers employ contact sliders or slip-rings and brushes as do resistively coupled transducers (potentiometer) which results in increased design life and low mechanical loading.
- In mutual-induction transducers, a change in the flux linkage is effected by one of two common techniques.
    - One technique is to ***move an object made of ferromagnetic material within the flux path between the primary coil and the secondary coil.***
    - The other common way to ***change the flux linkage is to move one coil with respect to the other.***
    - Motion can be measured by using the secondary signal (i.e., induced voltage in the secondary coil).

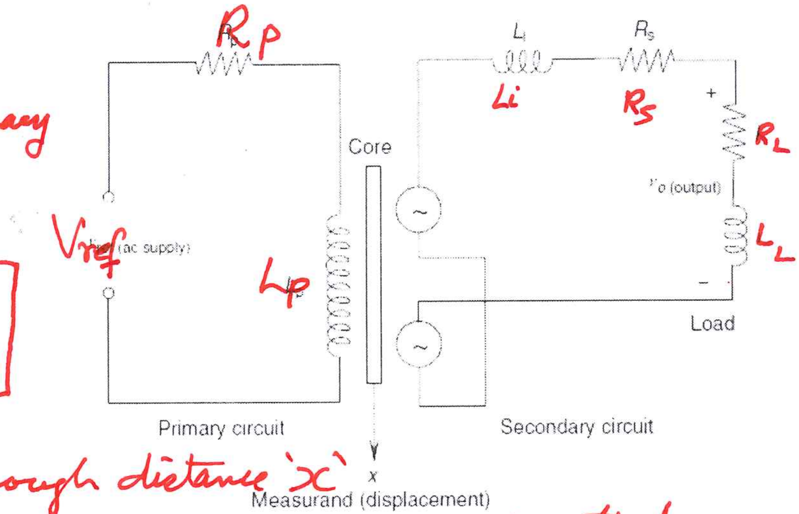
Linear-Variable Differential Transformer (LVDT)



- As the core moves, the reluctance of the flux path between the primary and the secondary coils changes.
- The ***degree of flux linkage*** depends on the axial position of the core.
- Since the two secondary coils are connected in series opposition, so that the potentials induced in the two secondary coil segments oppose each other, it is seen that the ***net induced voltage is zero when the core is centered between the two secondary winding*** segments. *This is known as the null position.* When the core is displaced from this position, a nonzero induced voltage is generated. At steady state, the amplitude $V_0 \propto$ Core displacement x in the linear (operating) region. Consequently, ***V_0 may be used as a measure of the displacement.***
- Note that because of opposed secondary windings, the LVDT provides the direction as well as the magnitude of displacement.



Linear-Variable Differential Transformer (LVDT) Equivalent Circuit.



Magnetizing Voltage in Primary Coil is given by:

$$V_p = V_{ref}\left[\frac{j\omega L_p}{R_p + j\omega L_p}\right]$$

*Suppose we move Core through distance 'x' from "Null Position."

CORE of length L_o

*Induced Voltage in ONE secondary segment

$$V_a = V_p K_a\left(\frac{L}{2} + x\right)$$

in the other

$$V_b = V_p K_b\left(\frac{L}{2} - x\right)$$

; K_a, K_b non-linear functions of Core position & Complex functions of frequency variable (x).

* Due to series Connection of two segments:

Secondary Voltage $V_s = V_a - V_b = V_p\left[K_a\left(\frac{L}{2}+x\right) - K_b\left(\frac{L}{2}-x\right)\right]$

*Ideal Case: $K_a(\cdot), K_b(\cdot)$ are equal $\therefore V_a = V_b$ and $V_s = 0$ May be ϕ-diff.

* If ϕ-difference; $K_a\left(\frac{L}{2}\right) - K_b\left(\frac{L}{2}\right)$ will have a small diff (Magnitude) but phase will be 90° w.r.t both $K_a + K_b$; This is Quadrature error.

FOR SMALL x, TAYLOR SERIES EXPANSION

$$V_s = V_p\left[K_a\left(\frac{L}{2}\right) + \frac{\partial K_a}{\partial x}\left(\frac{L}{2}\right)x - K_b\left(\frac{L}{2}\right) + \frac{\partial K_b}{\partial x}\left(\frac{L}{2}x\right)\right]$$

If $K_0 = K_a = K_b$ $\quad V_s = 2V_p \frac{\partial K_0}{\partial x}\left(\frac{L}{2}\right)x$;

or $\quad V_s = V_p K x$; where $K = 2\frac{\partial K_0}{\partial x}\left(\frac{L}{2}\right)$

In this Case Induced Voltage proportional to x is given by:

$$v_s = v_{ref}\left[\frac{j\omega L_p}{R_p + j\omega L_p}\right] Kx$$

Output Voltage v_o at Load is given by:

$$v_o = \left[\frac{j\omega L_p}{R_p + j\omega L_p}\right]\left[\frac{R_L + j\omega L_L}{(R_L + R_s) + j\omega(L_L + L_1)}\right] Kx$$

For Small displacements:

$$v_o\,(LVDT) \propto \text{displacement } x$$

ϕ-lead @ the output is given by:

$$\phi = 90^\circ - \tan^{-1}\frac{\omega L_p}{R_p} + \tan^{-1}\frac{\omega L_L}{R_L} - \tan^{-1}\frac{\omega(L_L + L_1)}{R_L + R_s}$$

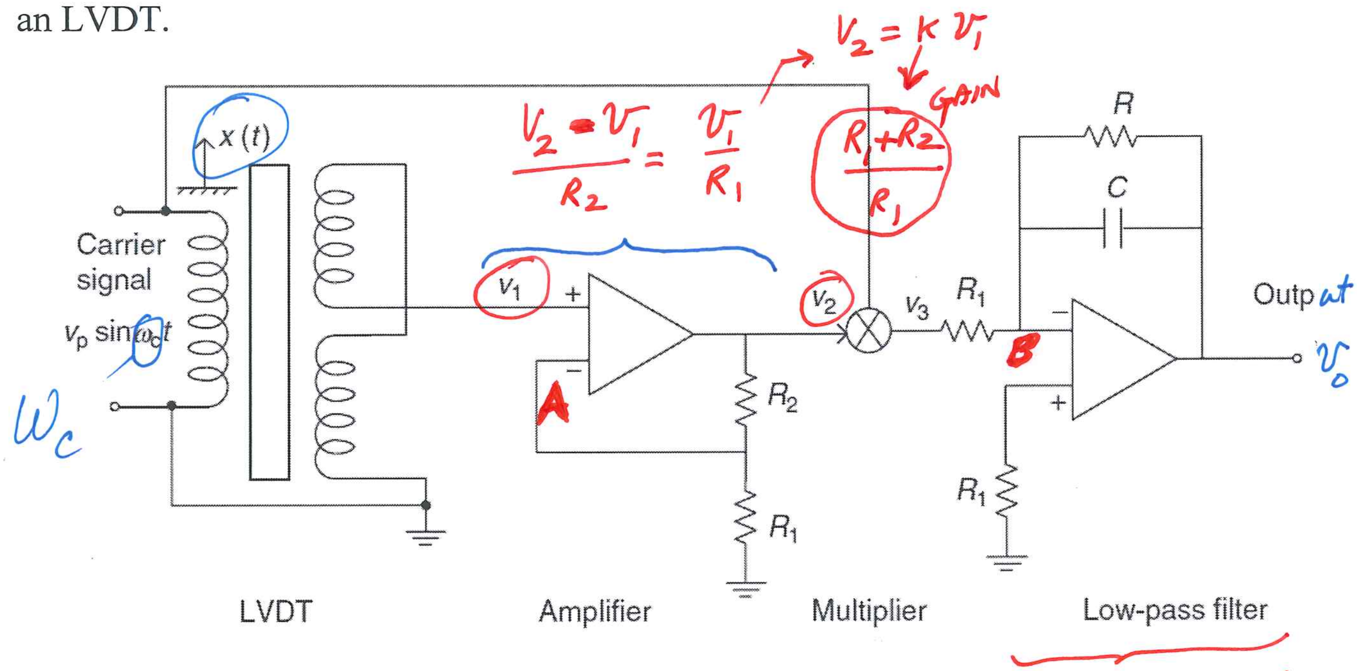
* Level of dependancy of ϕ-shifts on the load including Secondary Can be:

Reduced by increasing the Load Impedance.

Signal Conditioning:

- Signal Amplification – increase signal strength so we ca interpret it.
- Filtering – need exactly the signals we require for interpreting it properly.
- Improving SNR – filter out unwanted so actual signal quality is better and Noise (unwanted) signal is suppresses.

Example:
Figure shows a schematic diagram of a simplified signal-conditioning system for an LVDT.

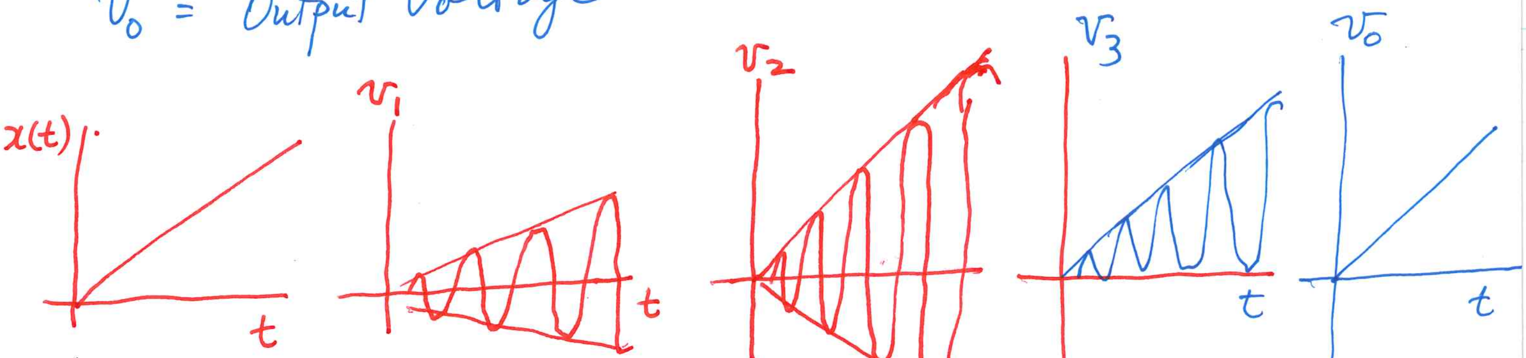


$x(t)$ = Displacement of the LVDT CORE

W_c = frequency of the Carrier Voltage

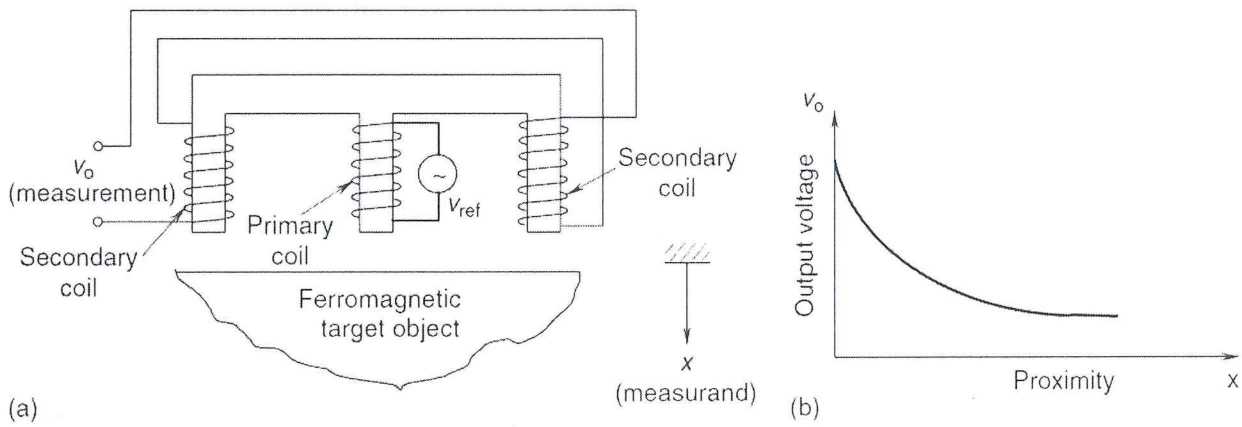
V_o = Output Voltage

$C \times R$ Signal will be filtered out by LPF



See LVDT Example for more details.

Mutual Induction Proximity Sensor:



- Displacement transducer also operates on the mutual-induction principle.

- The insulating ***E-shaped*** core carries the primary winding in its middle limb. The two end limbs carry secondary windings, which are ***connected in series***. Unlike the LVDT and the RVDT, the two voltages induced in the secondary winding segments are additive in this case.

- Proximity sensors are used in a wide variety of applications pertaining to non-contacting displacement sensing and dimensional gaging. Few applications are:

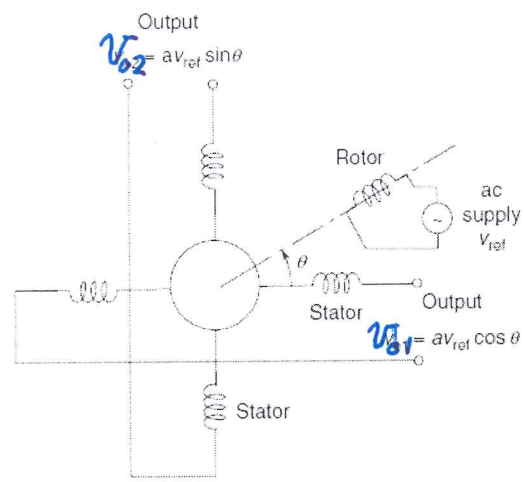
    - Measurement and control of the gap between a robotic welding torch head and the work surface.

    - Gaging the thickness of metal plates in manufacturing operations (e.g., rolling and forming).

    - Angular speed measurement at steady state, by counting the number of rotations per unit time

    - Level detection (e.g., in the filling, bottling, and chemical process industries)

Resolver: This mutual-induction transducer is widely used for measuring angular displacements.



- Rotor contains the primary coil & It consists of a ***single two-pole winding element*** energized by an ac supply voltage V_{ref}
- Rotor is directly attached to the object whose rotation is measured.
- ***Stator*** consists of two sets of windings placed 90^0 apart.
- If the angular position of the rotor with respect to one pair of stator windings is denoted by θ, the induced voltage in this pair of windings is given by:

$$V_{01} = a V_{ref} \cos\theta$$

Induced Voltage in other pair of Windings is given by

$$V_{02} = a V_{ref} \sin\theta$$

Amplitude Modulated Signals by V_{ref} (Carrier Signal)
Termed as Quadrature Signals.

a: dependant on geometry + Material characteristics of device.

e.g: ratio of # of TURNS IN ROTOR and STATOR WINDINGS.

Any: V_{01} or V_{02} can be used to determine angular position θ $[0, 90°]$

Need Both for displacement: Magnitude and Direction

Example: Angular position Direction $90+\theta$, $90-\theta$ give Same SINE
But $90+\theta$, $90-\theta$ Sign Will Diff for COSINE
$\therefore$ Proper Direction.

Demodulation

- As for differential transformers (i.e., LVDT and RVDT) transient displacement signals of a resolver can be extracted by demodulating its (modulated) outputs.
- This is accomplished by filtering out the carrier signal, thereby extracting the modulating signal.

Revisiting Two output signals from Resolver

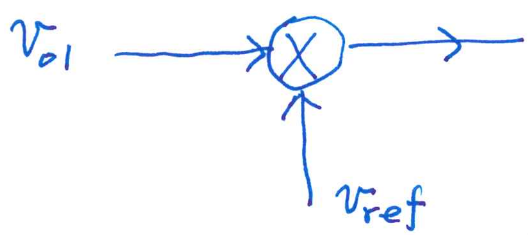
$$V_{o1} = a\,V_{ref}\cos\theta \;,\quad V_{o2} = a\,V_{ref}\sin\theta.$$

These are called Quadrature Signals

using V_{ref} (C_xR) Signal $= V_a \sin\omega t$

∴ Induced Signals will be:

$$V_{o1} = a\,V_a \cos\theta \sin\omega t$$
$$V_{o2} = a\,V_a \sin\theta \sin\omega t$$



$$V_{m1} = V_{o1}\,V_{ref}$$
$$V_{m2} = V_{o2}\,V_{ref}$$

→ $V_{m1} = V_{o1} * V_{ref} = a\,V_a^2 \cos\theta \sin^2\omega t = \frac{1}{2} a V_a^2 \cos\theta(1-\cos 2\omega t)$

→ $V_{m2} = V_{o2} * V_{ref} = a\,V_a^2 \sin\theta \sin^2\omega t = \frac{1}{2} a V_a^2 \sin\theta(1-\cos 2\omega t)$

→ $\omega_c : 2\pi f_c$: is $10 \times$ Max. frequency Content - in. displacement θ

use LPF with Cutoff $\omega/10$ and V_{m1} + V_{m2}

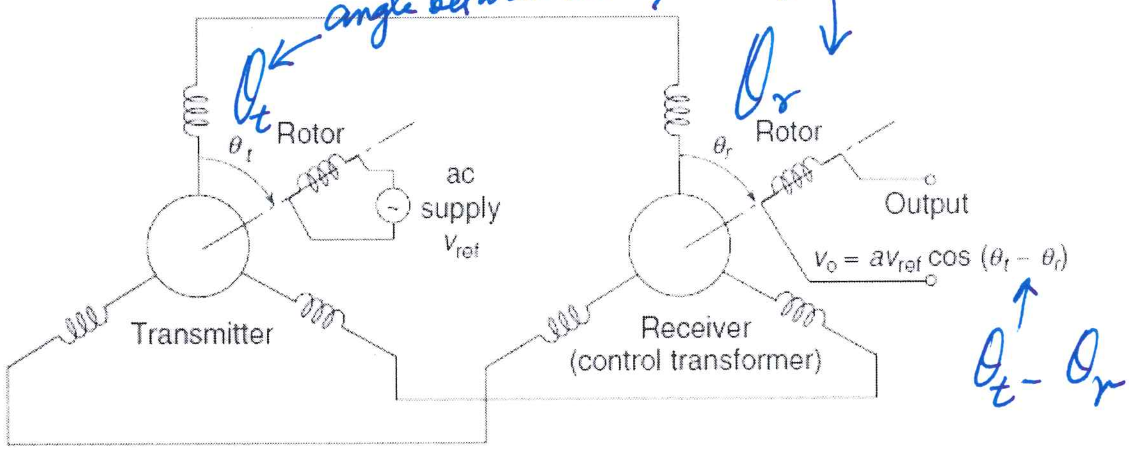
∴ $V_{f_1} = \frac{1}{2} a V_a^2 \cos\theta$ and $V_{f_2} = \frac{1}{2} a V_a^2 \sin\theta.$

Providing $\cos\theta$ + $\sin\theta$ ⇒ Magnitude and Sign of θ.

Sign of θ ⇒ direction

Synchro Transformer:
The "synchro" is somewhat similar in operation to the resolver. The main differences are that the synchro employs two identical rotor–stator pairs, and each stator has three sets of windings, which are placed 120^0 apart around the rotor shaft.

angle between Rotor & Winding



- Both Rotors have single ø windings
- One of the Rotors is energized with 'ac' V_{ref}
  - → This induces voltage in 3 winding segments of corr. stator.
  - → 3 diff. Amplitudes → depending on angular pos. of ROTOR.
  - → This DRIVE ROTOR-STATOR is called Transmitter.
- Other ROTOR-STATOR is called: Receiver
- Windings of Tx STATOR now connected to Rx STATOR windings
  - This induces a voltage V_o in Rx ROTOR

→ If $\theta_t = \theta_r \Rightarrow$ Receiver ROTOR is aligned with direction

- $V_o = a V_{ref} \cos(\theta_t - \theta_r)$ will be MAX.

- Synchros are operated near $\theta_r = \theta_t + 90^\circ$; $V_o = 0$ at this pt.

So we define a New angle $\theta \therefore \theta_r = (\theta_t + 90) - \theta$

$\therefore V_o = a V_{ref} \sin(\theta)$

θ: relative displacement between 2 Rotating objects.