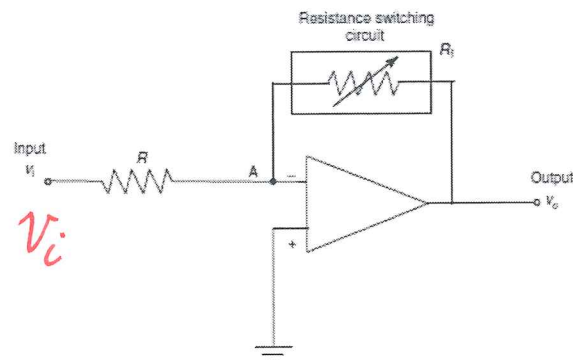
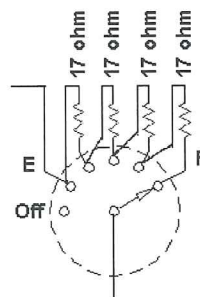


Curve Shaping Circuitry:

- Sort of like an amplifier with adjustable gain.
- Adjustable Feedback resistor R_f
- Bank of resistors and automatic switching can be deployed using Zener diodes.



- Amplifier Gain Variable in discrete steps
- One can use a potentiometer as R_f (feed back resistor) so the gain can be adjusted continuous
- Various Resistor steps \Rightarrow Diff V



Looking at Node -A.

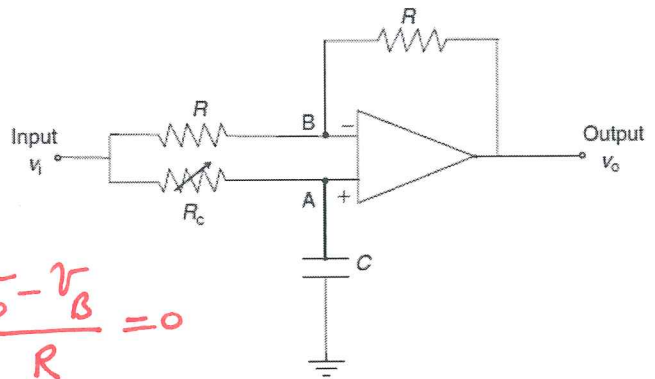
$$\frac{V_i}{R} + \frac{V_o}{R_f} = 0$$

$$V_o = - \frac{R_f}{R} V_i$$

$$\therefore V_o \propto R_f \quad (\text{ignore -ve sign})$$

Phase Shifters:

Phase Angle $\propto R_c$?
Starting with Nodes A & B.



Node A

$$\frac{v_i - v_A}{R_c} = \frac{C dv_A}{dt}$$

$$v_i = v_A + R_c C \frac{dv_A}{dt}$$

Laplace !!

$$v_i = v_A + R_c C v_A s$$

$$v_i = v_A (1 + \tau s) \quad \text{--- (1)}$$

Node B

$$\frac{v_i - v_B}{R} + \frac{v_o - v_B}{R} = 0$$

$$\frac{v_i}{R} - \frac{v_B}{R} + \frac{v_o}{R} - \frac{v_B}{R} = 0$$

$$v_B = \frac{1}{2} (v_i + v_o) \quad \text{--- (2)}$$

In op amp $v_A = v_B$; Sub (2) into (1)

$$v_i = \frac{1}{2} (v_i + v_o) (1 + \tau s); \quad \tau = R_c C : \text{Time Constant}$$

Transfer function $G(s)$ can be evaluated as follows:

$$\frac{v_o}{v_i} = G(s) = \frac{1 - \tau s}{1 + \tau s}$$

$$\text{Magnitude } |G(j\omega)| = \frac{\sqrt{1 + \tau^2 \omega^2}}{\sqrt{1 + \tau^2 \omega^2}} = 1.$$

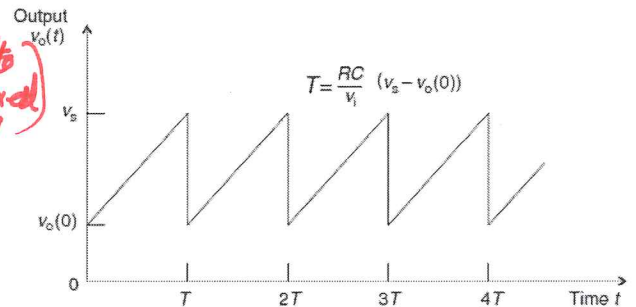
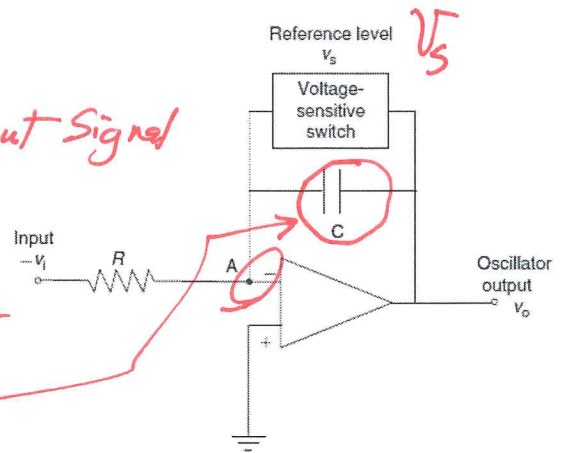
$$\begin{aligned} \text{Phase Angle } \angle G(j\omega) &= -\tan^{-1} \tau \omega - \tan^{-1} \tau \omega \\ &= -2 \tan^{-1} \tau \omega \\ &= -2 \tan^{-1} R_c C \omega \end{aligned}$$

\therefore Phase Angle $\propto R_c$

Magnitude = 1 \Rightarrow Undistorted; Angle being -ve \Rightarrow phase lag.
and Phase lag $\propto R_c$

Voltage to Frequency Converter: (VFC)

- VFC produces a periodic output signal
output signal $f \propto$ level of V_i
- Also called VCO: Voltage Controlled Oscillator
- Most Common: use a CAP.
- Time required for CAP to charge (to fixed V) is inversely proportional to changing voltage.



Node-A
$$\frac{V_i}{R} = C \frac{dv_o}{dt}$$

$$v_o(t) = \frac{1}{RC} v_i(t) + v_o(0)$$

Switch closes when $v_o(t) = V_s$ (ref); CAP discharges
CAP charging time T is given by

$$V_s = \frac{1}{RC} v_i T + v_o(0)$$

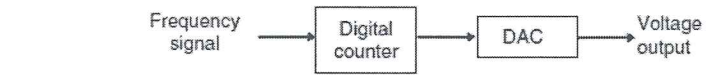
$$\therefore T = \frac{RC}{v_i} (V_s - v_o(0))$$

Switch opens again to charge / closes to discharge
 \Rightarrow sawtooth wave

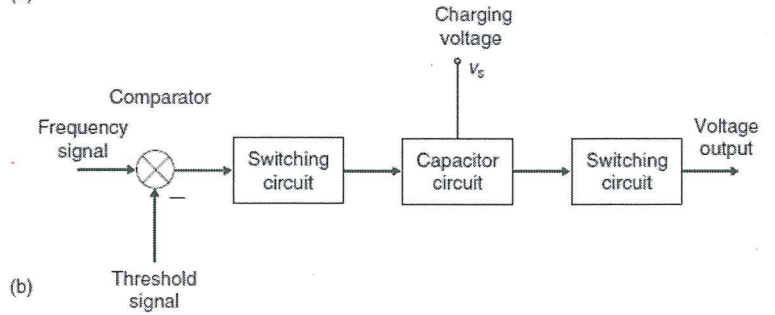
$$f = \frac{v_i}{RC(V_s - v_o(0))}$$

Frequency to Voltage Convertor

* Use digital counter to count signal frequency and use DAC to obtain voltage proportional to frequency



* Frequency signal supplied to comparator with threshold voltage level.



- output depends on which level is higher or lower than other: (Sign)
- First sign change (- to +) in comparator will trigger switching and connect to charge CAP.
- Next (+ to -) change will short CAP & discharge.
- level of charge is dependant on switching period.
- used for demodulation

Voltage to Current Convertor:

Node-A

$$\frac{V_A}{R} = \frac{V_P - V_A}{R}$$

Node-B

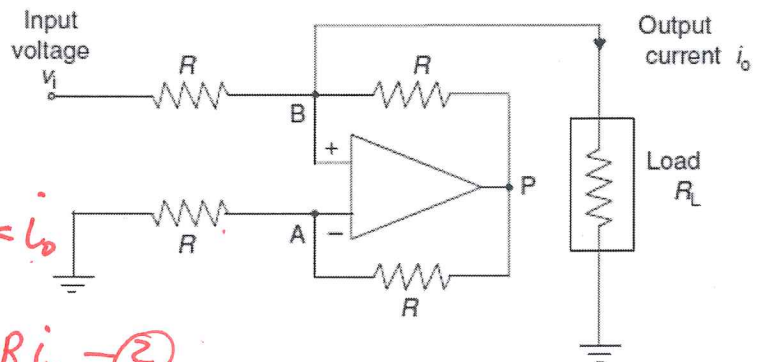
$$\frac{V_i - V_B}{R} + \frac{V_P - V_B}{R} = i_o$$

$$\Rightarrow 2V_A = V_P \quad \text{--- (1)} \quad \Rightarrow V_i - 2V_B + V_P = R i_o \quad \text{--- (2)}$$

Using fact $V_A = V_B$
sub (1) into (2)

$$i = \frac{V_i}{R}$$

$$\therefore i \propto V_i$$

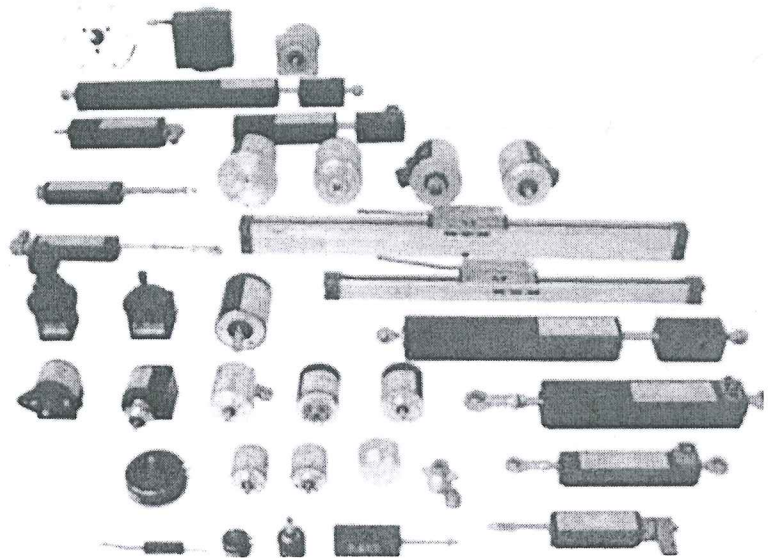


Motion Transducers:

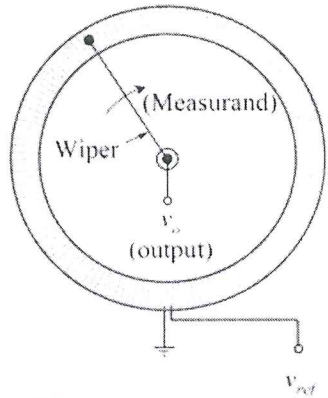
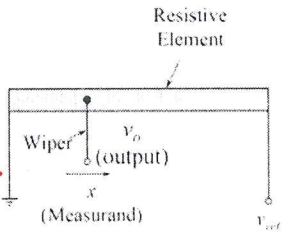
- Potentiometers (resistively coupled)
- Variable inductance (electromagnetically coupled)
- Variable capacitance
- Eddy current
- Piezoelectric

Potentiometer:

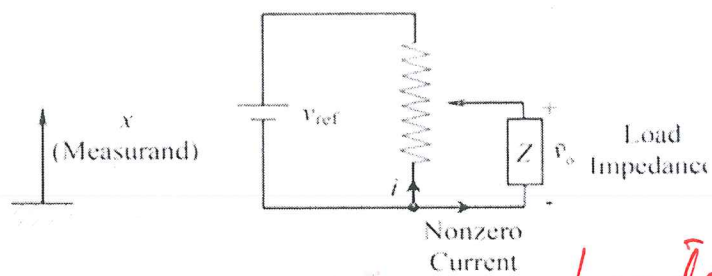
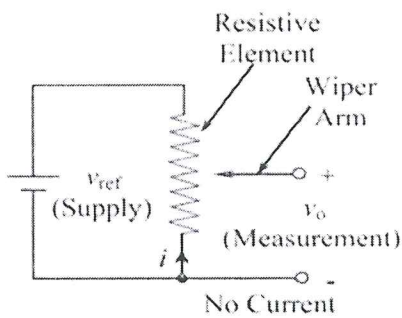
- Uniform coil of wire or a film of high resistive material- Carbon, platinum, or conductive plastic.



Rectilinear



Angular Motions

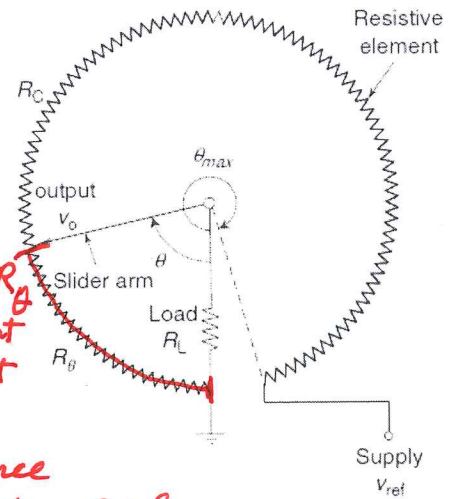


Loading

As load is connected Vref will change?

Loading Nonlinearity:

- What is the significance of the electrical loading nonlinearity error caused by a purely resistive load connected to the pot?



- For general position θ of Arm, let $R = R_\theta$ in Output Segment
- Assuming Coil is Uniform

$$\therefore R_\theta = \frac{\theta}{\theta_{max}} R_C \leftarrow \text{Total Resistance of Potentiometer Coil}$$

- Current Balance @ Sliding Node (pt.) is:

from here multiply by R_C + use ①

$$\frac{V_{ref} - V_o}{R_C - R_\theta} = \frac{V_o}{R_\theta} + \frac{V_o}{R_L} \leftarrow \text{Load Resistor}$$

$$\frac{V_{ref} - V_o}{1 - \frac{R_\theta}{R_C}} = \frac{V_o}{\frac{R_\theta}{R_C}} + \frac{V_o}{\frac{R_L}{R_C}}$$

$$\therefore \frac{V_{ref} - V_o}{1 - \frac{\theta}{\theta_{max}}} = \frac{V_o}{\frac{\theta}{\theta_{max}}} + \frac{V_o}{R_L/R_C}$$

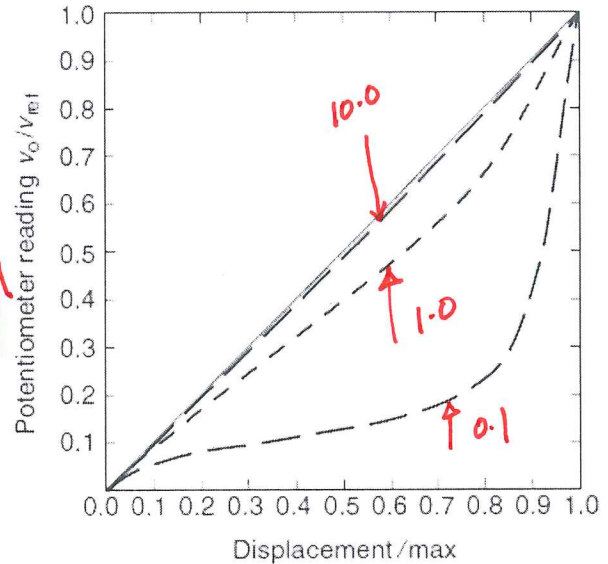
$$\frac{V_{ref}}{\theta_{max} - \theta} = \frac{V_o}{\theta_{max} - \theta} + \frac{V_o}{\frac{\theta}{\theta_{max}}} + \frac{V_o}{R_L/R_C}$$

Applying Algebra:

$$\frac{V_o}{V_{ref}} = \left[\frac{(\theta/\theta_{max})(R_L/R_C)}{(R_L/R_C + (\theta/\theta_{max})) - (\theta/\theta_{max})^2} \right]$$

$$\frac{V_o}{V_{ref}} = \left[\frac{(\theta/\theta_{max}) (R_L/R_C)}{\left(\frac{R_L}{R_C} + (\theta/\theta_{max}) - \frac{\theta}{\theta_{max}} \right)^2} \right]$$

- High Loading Error for Low $\frac{R_L}{R_C}$ ratio.
- Good Accuracy for $\frac{R_L}{R_C} > 10$
- Small values of θ/θ_{max}



- - - - $R_L/R_C = 0.1$
 $R_L/R_C = 1.0$
 - - - - $R_L/R_C = 10.0$

What to do to reduce loading Error in Pots?

- Increase R_L/R_C : Load Impedance V_s Coil Impedance.
- Use Pots to measure small values of θ/θ_{max} ;
Calibrate Only a small segment of Resistance element for Linear reading

Loading - non-linearity is given by

$$e = \frac{\left(\frac{V}{V_{ref}} - \frac{\theta}{\theta_{max}} \right)}{\frac{\theta}{\theta_{max}}} * 100\%$$

$e @ \theta/\theta_{max} = 0.5$

$\frac{R_L}{R_C}$	Error (%)
0.1	-71.4%
1.0	-20%
10.0	-2.4%

Resolution: defined by # of turns in COIL.
Coil of N turns, resolution $r = \frac{100}{N} \%$

Example:

A high-precision mobile robot uses a potentiometer attached to the drive wheel to record its travel during autonomous navigation. The required resolution for robot motion is 1 mm, and the diameter of the drive wheel of the robot is 20 cm. Examine the design considerations for a standard (single-coil) rotatory potentiometer to be used in this application.

$$1 \text{ mm} = 0.1 \text{ cm}; \text{ Diameter} = 20 \text{ cm} \Rightarrow r = 10 \\ \text{Circumference} = 2\pi(10) = 20\pi$$

Solution:

• Assuming POT is directly connected to drive wheel.

• Required POT resolution; $\frac{0.1 \text{ cm}}{20\pi} \times 100\% = 0.16\%$

• using $\sigma = \frac{100}{N}\% \Rightarrow N = \frac{100}{0.16} = 625 \text{ turns}$

• Pot Dia = 10 cm; Wire diameter = d

• Potentiometer Circumference = $\pi \times 10 = 625 \times d$

$$\therefore d = 0.5 \text{ mm}$$

• Assume POT resistance = 5 Ω

Wire Resistivity = 4 $\mu\Omega/\text{cm}$

$$\text{Diameter of Core} = \frac{4 \times 10^{-6} \times \pi D \times 625}{\pi (0.05/2)^2} = 5 \Omega$$

• Resistivity = resistance \times (cross-sectional Area) / length

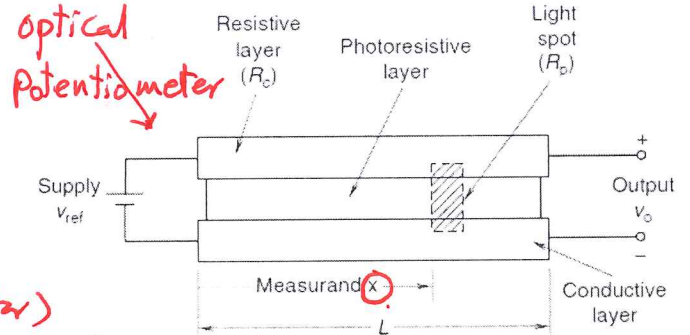
$$\therefore D = 1.25 \text{ cm}$$

• Potentiometer Sensitivity $S = \frac{\Delta v_o}{\Delta \theta}$; $\frac{\text{Change in } v_o}{\text{Change in } \theta}$

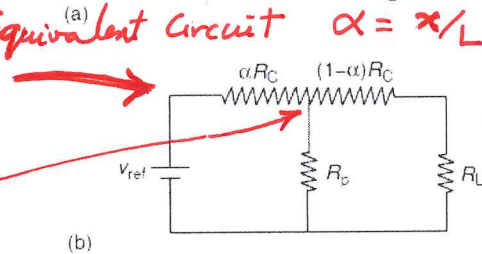
Read Example 4.2 (first Edition) Text.

Optical Potentiometer:

The optical potentiometer, shown is a displacement sensor. A layer of photoresistive material is sandwiched between a layer of ordinary resistive material and a layer of conductive material.



* At equivalent ckt, R_L (Load Resistor) is present at the output of POT



* Current through Load V_o/R_L

* $\alpha = \frac{x}{L}$; Fractional Position of light spot

* Current balance @ Junction:

$$\frac{V_{ref} - [(1-\alpha)R_c + R_L] \frac{V_o}{R_L}}{\alpha R_c} = \frac{V_o}{R_L} + \frac{[(1-\alpha)R_c + R_L] \frac{V_o}{R_L}}{R_p}$$

Can be re-written as:

$$\frac{V_o}{V_{ref}} \left\{ \frac{R_c}{R_L} + 1 + \frac{x}{L} \frac{R_c}{R_p} \left[\left(1 - \frac{x}{L}\right) \frac{R_c}{R_L} + 1 \right] \right\} = 1$$

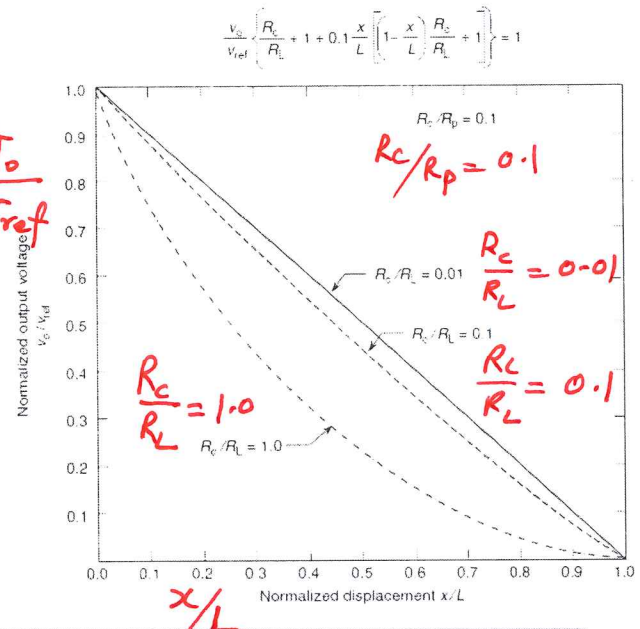
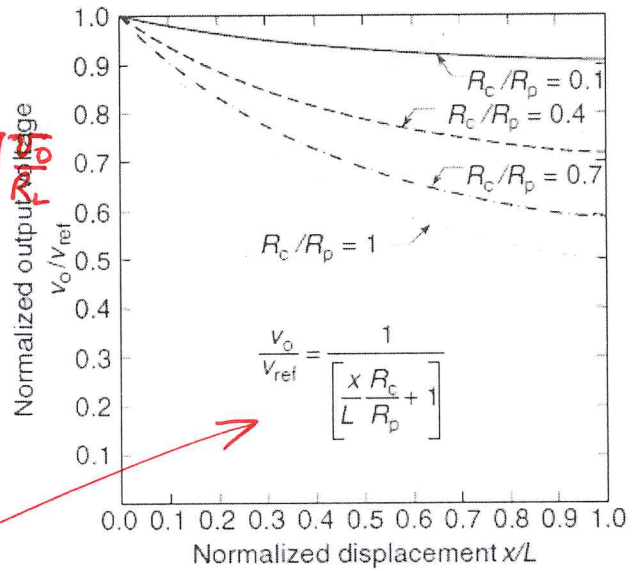
if $R_L \gg R_c$; $R_c/R_L \approx 0$

$$\therefore \frac{V_o}{V_{ref}} = \frac{1}{\left[\frac{x}{L} \cdot \frac{R_c}{R_p} + 1 \right]}$$

→ Original Equation is plotted →

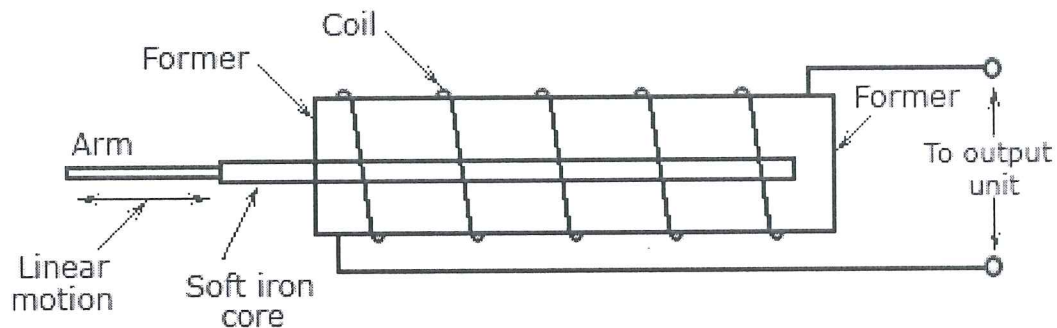
$$\frac{V_o}{V_{ref}}$$

* Behaviour of optical POT becomes more linear for higher values of load resistance



Variable Inductance Transducers:

- When the flux linkage (defined as magnetic flux density times the number of turns in the conductor) through an electrical conductor changes, *a voltage in proportion to the rate of change of flux* is induced in the conductor.



www.InstrumentationToday.com

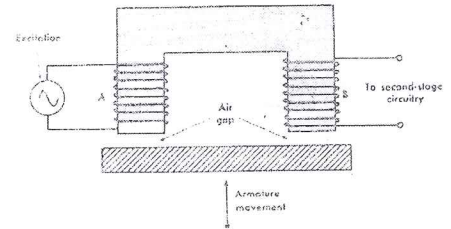
- This voltage in turn, generates a magnetic field, which opposes the original (primary) field. Hence, a mechanical force is necessary to sustain the change of flux linkage.
- If the change in flux linkage is brought about by a relative motion, the associated mechanical energy is directly converted (induced) into electrical energy.
- This is the basis of electromagnetic induction, and it is the principle of operation of electrical generators and variable-inductance transducers.
- *The induced voltage or change in inductance may be used as a measure of the motion.*

Three primary types can be identified as:

- Mutual-induction transducers
- Self-induction transducers
- Permanent-magnet transducers.

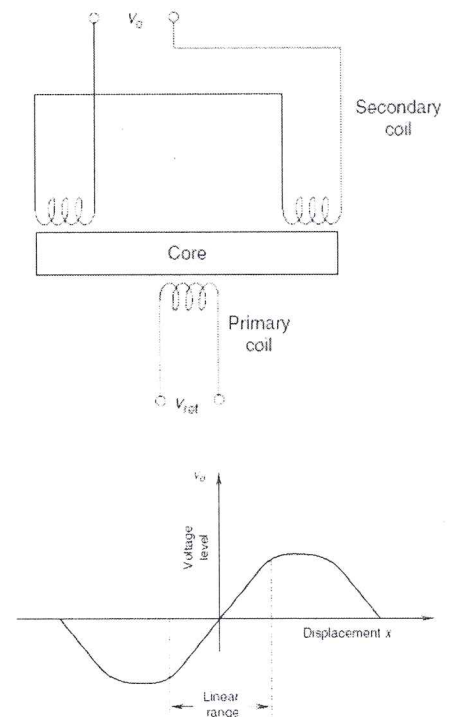
Mutual-induction transducers:

- Arrangement of a mutual-induction transducer constitutes two coils, the primary winding and the secondary winding. One of the coils (primary winding) carries an alternating-current (ac) excitation, which induces a steady ac voltage in the other coil (secondary winding).
- The level (amplitude, rms-value, etc.) of the induced voltage depends on the flux linkage between the coils.
- None of these transducers employ contact sliders or slip-rings and brushes as do resistively coupled transducers (potentiometer) which results in increased design life and low mechanical loading.
- In mutual-induction transducers, a change in the flux linkage is effected by one of two common techniques.
 - One technique is to *move an object made of ferromagnetic material within the flux path between the primary coil and the secondary coil.*
 - The other common way to *change the flux linkage is to move one coil with respect to the other.*
 - Motion can be measured by using the secondary signal (i.e., induced voltage in the secondary coil).



Linear-Variable Differential Transformer (LVDT)

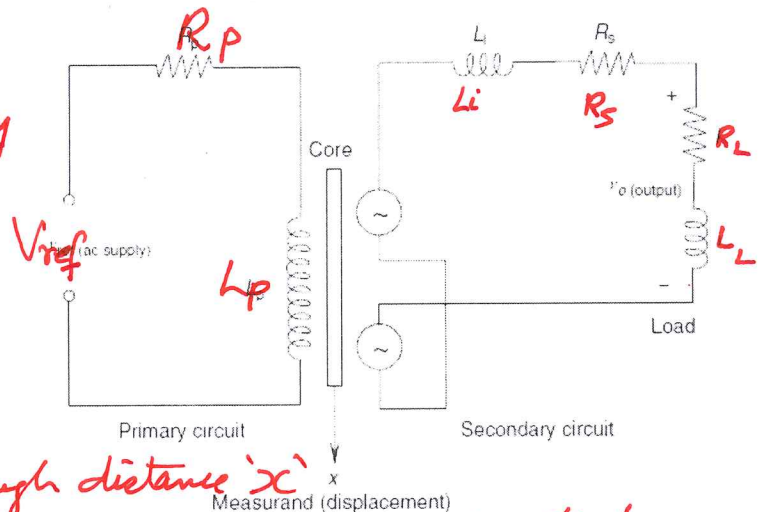
- As the core moves, the reluctance of the flux path between the primary and the secondary coils changes.
- The *degree of flux linkage* depends on the axial position of the core.
- Since the two secondary coils are connected in series opposition, so that the potentials induced in the two secondary coil segments oppose each other, it is seen that the *net induced voltage is zero when the core is centered between the two secondary winding segments. This is known as the null position.* When the core is displaced from this position, a nonzero induced voltage is generated. At steady state, the amplitude $V_0 \propto$ Core displacement x in the linear (operating) region. Consequently, V_0 *may be used as a measure of the displacement.*
- Note that because of opposed secondary windings, the LVDT provides the direction as well as the magnitude of displacement.



Linear-Variable Differential Transformer (LVDT) Equivalent Circuit.

Magnetizing Voltage in Primary Coil is given by:

$$V_p = V_{ref} \left[\frac{j\omega L_p}{R_p + j\omega L_p} \right]$$



* Suppose we move Core through distance 'x' from "Null Position". CORE of length l

* Induced Voltage in ONE secondary segment

$$V_a = V_p K_a \left(\frac{l}{2} + x \right)$$

in the other

$$V_b = V_p K_b \left(\frac{l}{2} - x \right); \quad K_a, K_b \text{ non-linear functions of Core position \& complex functions of frequency variable } (x).$$

* Due to series connection of two segments:

$$\text{Secondary Voltage } V_s = V_a - V_b = V_p \left[K_a \left(\frac{l}{2} + x \right) - K_b \left(\frac{l}{2} - x \right) \right]$$

* Ideal Case: $K_a(\cdot), K_b(\cdot)$ are equal $\therefore V_a = V_b$ and $V_s = 0$ Maybe ϕ -diff.

* If ϕ -difference; $K_a(\frac{l}{2}) - K_b(\frac{l}{2})$ will have a small diff (Magnitude) but phase will be 90° w.r.t both $K_a + K_b$; This is Quadrature error.

FOR SMALL x , TAYLOR SERIES EXPANSION

$$V_s = V_p \left[K_a \left(\frac{l}{2} \right) + \frac{\partial K_a}{\partial x} \left(\frac{l}{2} \right) x - K_b \left(\frac{l}{2} \right) + \frac{\partial K_b}{\partial x} \left(\frac{l}{2} \right) x \right]$$

$$\text{if } K_a = K_b = K_0 \quad V_s = 2 V_p \frac{\partial K_0}{\partial x} \left(\frac{l}{2} \right) x;$$

$$\text{or } V_s = V_p K x; \quad \text{where } K = 2 \frac{\partial K_0}{\partial x} \left(\frac{l}{2} \right)$$

In this case Induced voltage proportional to x is given by:

$$V_s = V_{ref} \left[\frac{j\omega L_p}{R_p + j\omega L_p} \right] Kx$$

Output voltage V_o at Load is given by:

$$V_o = \left[\frac{j\omega L_p}{R_p + j\omega L_p} \right] \left[\frac{R_L + j\omega L_L}{(R_L + R_S) + j\omega(L_L + L_1)} \right] Kx$$

For Small displacements:

$$V_o \text{ (LVDT)} \propto \text{displacement } x$$

ϕ -lead @ the output is given by:

$$\phi = 90^\circ - \tan^{-1} \frac{\omega L_p}{R_p} + \tan^{-1} \frac{\omega L_L}{R_L} - \tan^{-1} \frac{\omega(L_L + L_1)}{R_L + R_S}$$

* Level of dependency of ϕ -shifts on the load including Secondary can be:

Reduced by increasing the Load Impedance.