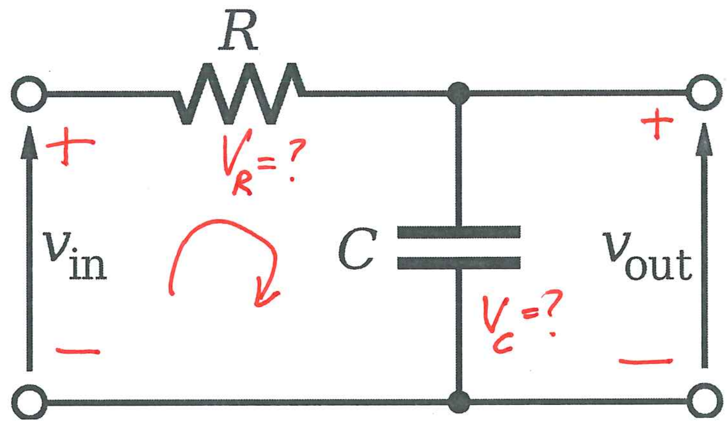
# Low Pass Filter with a cut-off:




Given a voltage @ input

step or continuous wave.

For Resistor Load. ohm's Law $V = IR$

$V_R = I_R R$

Across Capacitor Current $I = C\frac{dv}{dt}$.

$V_C$ across Capacitor

$\therefore I_C = C\frac{dV_C}{dt}$

using Voltage Load $V_R \rightarrow$ Resistor, $V_C \rightarrow$ Capacitor.

$\underbrace{I_R = I_C}$ and $\underbrace{V_{in} = V_R + V_C}$

$\frac{V_R}{R} = C\frac{dV_C}{dt}$ and using $V_R = V_{in} - V_C$

sub

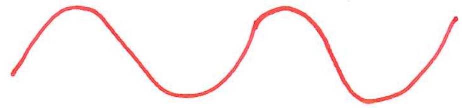
$\frac{V_{in} - V_C}{R} = \frac{C\,dV_C}{dt}$ ; taking $-\frac{V_C}{R}$ to Right.

$C\frac{dV_C}{dt} + \frac{V_C}{R} = \frac{V_{in}}{R}$

$\frac{dV_C}{dt} + \frac{V_C}{RC} = \frac{V_{in}}{RC}$ ; But $V_C = V_{out}$

$$\boxed{\frac{dv_o}{dt} + \frac{1}{RC}v_o = \frac{1}{RC}V_{in}}$$

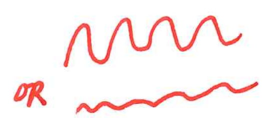
Let's say $V_{in} = V^* \sin \omega t$



$$V_o = A \cos \omega t + B \sin \omega t.$$

$$= -\frac{\omega RC}{1+(\omega RC)^2} V^* \cos \omega t + \frac{1}{1+(\omega RC)^2} V^* \sin \omega t$$

Look @ Magnitude of output. 

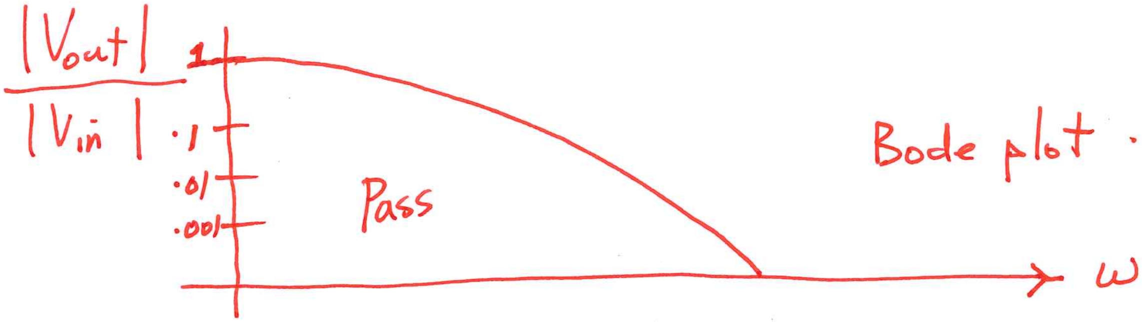



$$|V_{out}| = \frac{1}{1+\omega^2 (RC)^2} V^* = \frac{1}{1+\omega^2(RC)^2} |V_{in}| \, ;$$

Also write $\left|\frac{V_o}{V_i}\right| = \frac{1}{\tau s+1}$

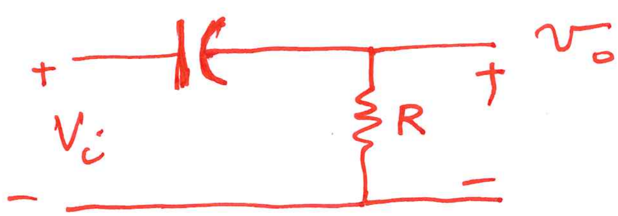
check $\omega$ is small ; $V_o$ is almost 1.

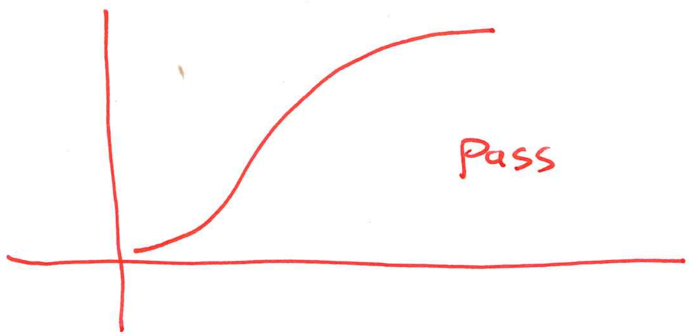
$\omega$ is high ; $V_o$ is almost $\phi$.

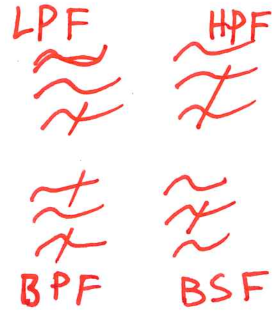



Bode plot.

High Pass:

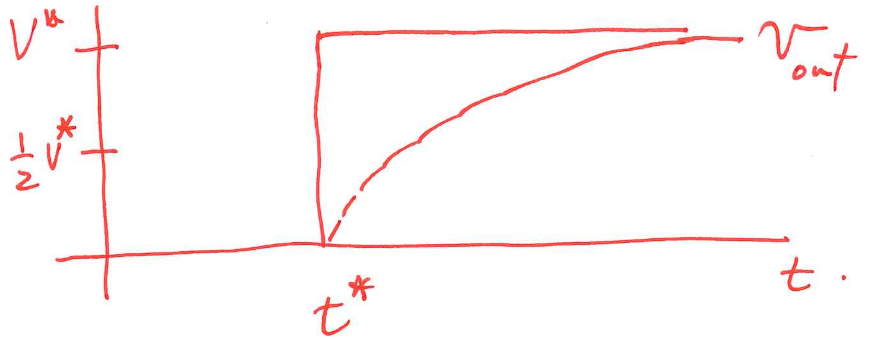


What if you applied a Step Function to this filter?

What would $V_0$ look like?



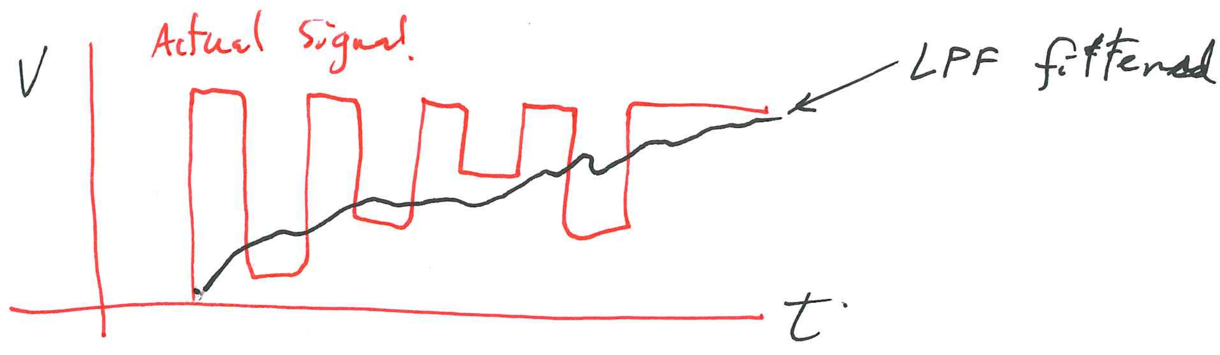

$$V_0 = V^*\left[1 - e^{-\frac{1}{RC}(t-t^*)}\right]$$

If we wanted to see how long it took to get to $\frac{1}{2}V^*$, we do the following.

$$V_0 = V^*\left[1 - e^{-1/RC\,(t-t^*)}\right] = \frac{1}{2} \Rightarrow e^{-\frac{1}{RC}(t-t^*)}$$

$$\therefore\ t - t^* = -RC\ \ln\left(\tfrac{1}{2}\right) = 0 = 0.637.\ (RC)$$

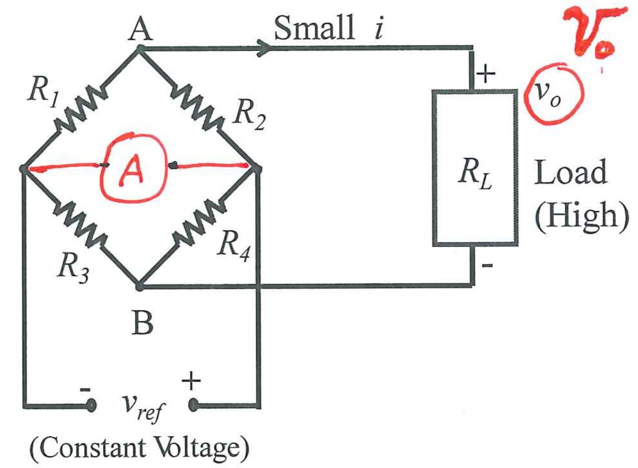
If RC gets Larger System Responds slower
RC " smaller " " faster.



## Bridge Circuits:

Bridge circuits are used to make a form of measurement:

- Change in resistance
- Change in inductance
- Change in capacitance
- Oscillating frequency




D.C. Bridge - All Resistors

A.C. Bridge - Impedances

* Bridge is Balanced iff A show No current flow

* when balanced

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad ; \text{ For Any } R_L$$

Look @ $V_o = V_A - V_B$

$$= \underbrace{\frac{R_1}{R_1+R_2} V_{ref}}_{V_A} - \underbrace{\frac{R_3}{R_3+R_4} V_{ref}}_{V_B}$$

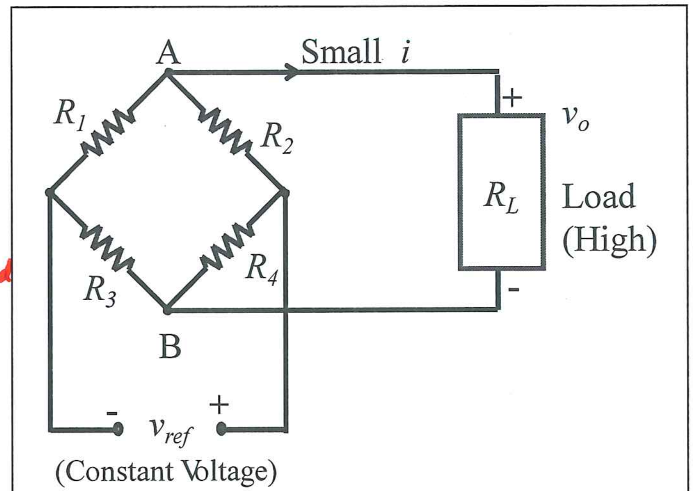
$$= V_{Ref}\left(\frac{\cancel{R_1R_3} + R_1R_4 - \cancel{R_3R_1} - R_3R_2}{(R_1+R_2)(R_3+R_4)}\right) = \frac{R_1R_4 - R_3R_2}{(R_1+R_2)(R_3+R_4)} V_{Ref}$$

Assume $R_1 = R_2 = R_3 = R_4 = R$. $\therefore$ Bridge is balanced. $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Let's increase $R_1$ by $\delta R$, all others @ $R$

$R \Rightarrow$ The only Active strain gage.

Continuiing.




$$\delta V_o = \frac{[(R+\delta R)R - R^2]}{(R+\delta R+R)(R+R)} v_{ref} - 0 \qquad (1)$$

when Balanced

$$\frac{\delta v_o}{v_{Ref}} = \frac{\cancel{R^2} + R\delta R - \cancel{R^2}}{4R^2 + 2R\delta R}$$

$$= \frac{R\delta R}{4R^2 + 2R\delta R}$$

$$= \frac{\delta R}{4R + 2\delta R}$$

$$= \frac{\delta R/R}{4 + 2\frac{\delta R}{R}} \; ; \; \frac{\delta R}{R} : \text{Non Linear}$$

If we assume $\frac{\delta R}{R}$ is Very Small compared to 2; we get Linearized relation.

$$\frac{\delta V_o}{V_{Ref}} = \frac{\delta R}{4R} \; ; \; \frac{1}{4} \Rightarrow \text{Bridge Sensitivity}$$

~25% change

[Active Resistance $R \to R+\delta R$]

$\Rightarrow$ Bridge sensitivity is given by $\frac{\delta V_o}{\delta R}$

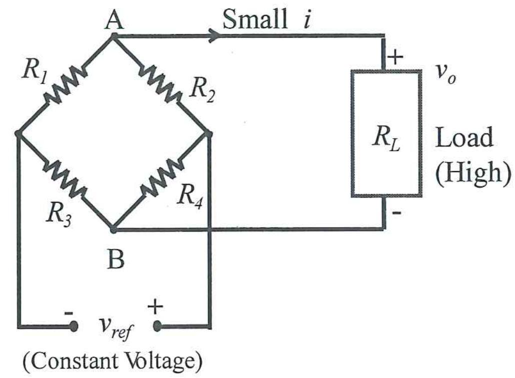
$$\therefore \frac{\delta V_o}{\delta R} = \frac{V_{ref}}{4R} \qquad (2)$$

$$N_p\,(\text{Non-Linearity \%ge}) = 100\left(1 - \frac{\text{Linearized output}}{\text{Actual output}}\right)\%$$

From EQ. (1) + (2) $N_p = 50\left(\frac{\delta R}{R}\right)\%$.

**Example:**

Suppose that in Figure on the right, at first $R_1 = R_2 = R_3 = R_4$. Now increase $R_1$ by $\delta R$, decrease $R_2$ by $\delta R$. This will represent two active elements that act in reverse, as in the case of two strain gage elements mounted on the top and the bottom surfaces of a beam in bending. Show that the bridge output is linear in $\delta R$ in this case.




**Solution:**

$R_1 = R+\delta R$, $R_2 = R-\delta R$

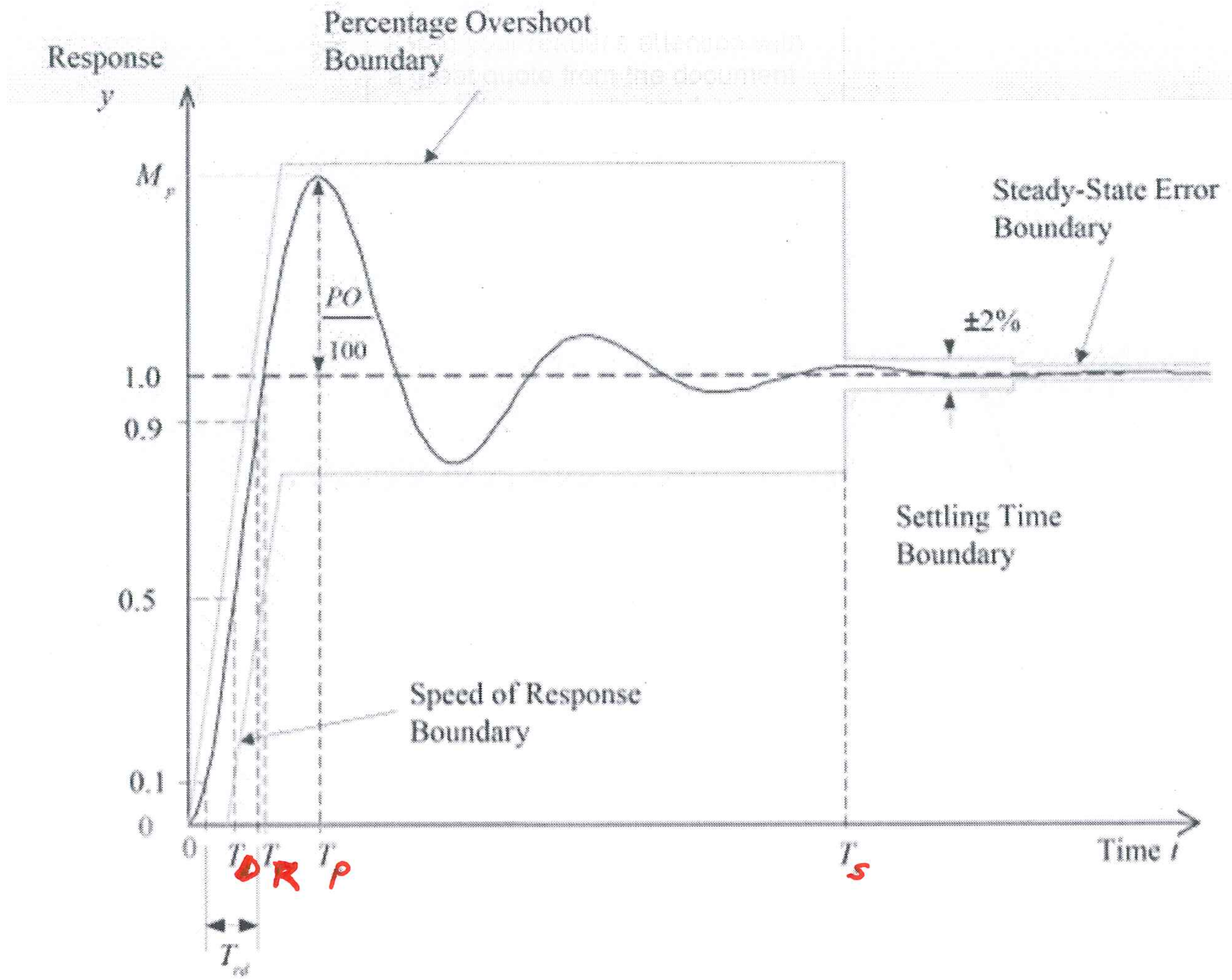
$$v_o = V_A - V_B = \frac{R+\delta R}{(R+\delta R)+(R+\delta R)} V_{ref} - \frac{R}{R+R} V_{ref} .$$

$$\delta v_o = \frac{(R+\delta R)\,2R - R(R+\delta R)(R-\delta R)}{(2R)(2R)} V_{ref}$$

$$\alpha\, v_o = \frac{2R^2 + 2\delta R - R(R^2-(\delta R)^2)}{4R^2} V_{ref}$$

$$=$$

$$\frac{\delta v_o}{V_{ref}} = \frac{\delta R}{2R} \quad ; \quad \cdot\, 0.5\%$$




Rise Time [$T_R$] : Time needed to pass 90% of steady state value, first time.

Delay Time [$T_D$] : Time needed to pass 50% of " " .

Peak Time [$T_P$]: Time for first Peak.

Setteling Time [$T_S$]: Time that the signal reaches ± 20% of steady state value and remains there.