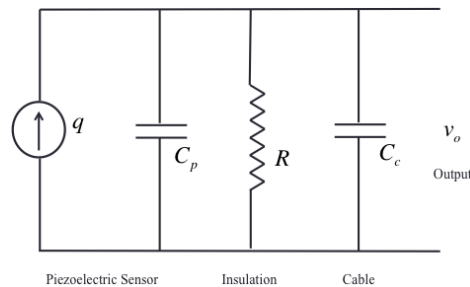


**P1-** A design objective in most control system applications is to achieve small time constants. An exception is the time constant requirement for a **piezoelectric sensor**. Explain why a large time constant, in the order of 1.0 s, is desirable for a piezoelectric sensor in combination with its signal-conditioning circuit. (**Hint:** You can use the results of the following paragraph to explain this).

An equivalent circuit for a piezoelectric accelerometer, which uses a quartz crystal as the sensing element, is shown in the Fig. P1. The generated charge is denoted by  $q$ , and the output voltage at the end of the accelerometer cable is  $v_o$ . The piezoelectric sensor capacitance is modeled by  $C_p$ , and the overall capacitance experienced at the sensor output, whose primary contribution is due to cable capacitance, is denoted by  $C_c$ . The resistance of the electric insulation in the accelerometer is denoted by  $R$ . Write a differential equation relating  $v_o$  to  $q$ . What is the corresponding transfer function? Using this result, show that the accuracy of accelerometer improves when the sensor time constant is large and when the frequency of the measured acceleration is high. For a quartz crystal with  $R = 1 \times 10^{11} \Omega$  and  $C_p = 300 \text{ pF}$ , and a circuit with  $C_c = 700 \text{ pF}$ , compute the time constant.



**Fig P1.**

**P2-** a. A standard accelerometer that weighs  $100 \text{ gm}$  is mounted on a test subject that has an equivalent mass of  $3 \text{ kg}$ . Estimate the accuracy in the first natural frequency of the object measured using this arrangement, considering mechanical loading due to accelerometer mass alone. If a miniature accelerometer that weighs  $0.5 \text{ gm}$  is used instead, what is the resulting accuracy?

b. A strain-gage accelerometer uses a semiconductor strain gage mounted at the roof of a cantilever element, with the seismic mass mounted at the free end of the cantilever. Suppose that the cantilever element has a square cross-section with dimensions  $1.5 \times 1.5 \text{ mm}^2$ . The equivalent length of the cantilever element is  $25 \text{ mm}$ , and the equivalent seismic mass is  $0.2 \text{ gm}$ . If the cantilever is made of an aluminum alloy with Young's modulus  $E = 69 \times 10^9 \text{ N/m}^2$ , estimate the useful frequency range of the accelerometer in Hertz.

**Hint:** When force  $f$  is applied to the free end of a cantilever, the deflection  $y$  at that location may be approximated by the formula

$$y = \frac{Fl^3}{3EI}$$

where  $l$  = cantilever length,

$I$  = second moment area of the cantilever cross-section about the bending axis =  $\frac{bh^3}{12}$ ,

$b$  = cross-section width,

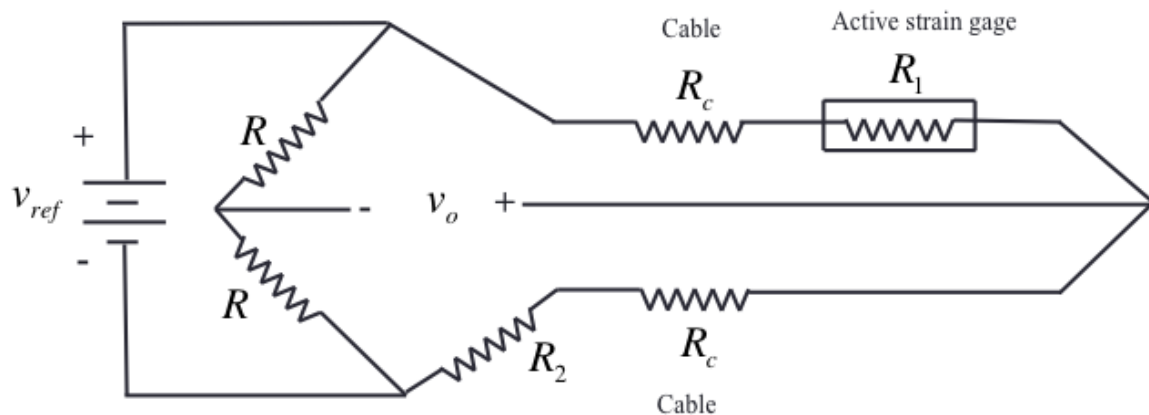
$h$  = cross-section height.

**P3-** Consider a standard strain-gage bridge with  $R_1$  as the only active gage and  $R_3 = R_4$ . Obtain an expression for  $R_1$  in terms of  $R_2$ ,  $v_o$ , and  $v_{ref}$ . Show that when  $R_1 = R_2$ , we get  $v_o = 0$  - a balanced bridge- as required. Note that the equation for  $R_1$ , assuming that  $v_o$  is measured using a high-impedance voltmeter, can be used to detect large resistance changes in  $R_1$ . Now suppose that the active gage  $R_1$  is connected to the bridge using a long, twisted wire pair, with each wire having a resistance of  $R_c$ . The bridge circuit has to be modified as in Figure P5 in this case.

Using the expression obtained earlier for  $R_1$ , show that the equation of the modified bridge is given by

$$R_1 = R_2 \left[ \frac{v_{ref} + 2v_o}{v_{ref} - 2v_o} \right] + 4R_c \frac{v_o}{[v_{ref} - 2v_o]}$$

Obtain an expression for the fractional error in the  $R_1$  measurement due to cable resistance  $R_c$ . Show that this error can be decreased by increasing  $R_2$  and  $v_{ref}$ .

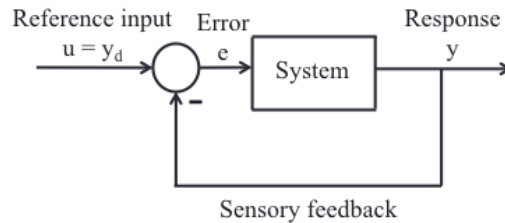


**Fig P3.**

**P4-** Describe methods of improving the displacement resolution and the velocity resolution in an encoder. An incremental encoder disk has 5000 windows. The word size of the output data is 12 bits. What is the angular (displacement) resolution of the device? Assume that quadrature signals are available but that no interpolation is used.

**P5-** Consider an encoder with 1000 windows in its track and capable of providing quadrature signals. What is the displacement resolution  $\Delta\theta_r$  in radians? Obtain a value for the nondimensional eccentricity  $\frac{e}{r}$  below which the eccentricity error has no effect on the sensor reading. For this limiting value, what is  $\frac{\Delta\theta_r}{(\frac{e}{r})}$ ? Typically, the values for this parameter, as given by encoder manufacturers, range from 3 to 6. Note:  $e$  = track eccentricity,  $r$  = track radius.

**P6-** Suppose that a feedback control system (figure below) is expected to provide an accuracy within  $\pm\Delta y$  for a response variable  $y$ . Explain why the sensor that measures  $y$  should have a resolution of  $\pm(\frac{\Delta y}{2})$  or better for this accuracy to be possible. An x-y table has a travel of 2 m. The feedback control system is expected to provide an accuracy of  $\pm 1$  mm. An optical encoder is used to measure the position for feedback in each direction (x and y). What is the minimum bit size that is required for each encoder output buffer? If the motion sensor used is an absolute encoder, how many tracks and how many sectors should be present on the encoder disk?



**Fig P6.**