Cmpt-405/705 March 11, 2016

Assignment 7

Problem 1

This problem deals with the BIN PACKING PROBLEM. Assume a ship arrives with n containers of weight $w_1, \ldots, w_n$ (all integers). Standing on the dock is a set of trucks, each of which can hold several containers with a total K units of weight (K is an integer, too). The goal is to minimise the number of trucks that are needed to carry all containers. A greedy algorithm you might use is the following.

Start with an empty truck, and begin piling containers $1, 2, 3, \ldots$ *into it until you get to a container that would overflow the weight limit. The truck is now loaded and you continue with a new truck.*

1. Give an example of a set of weights, and a value of K, where this algorithm does not use the minimum possible number of trucks.
2. Show, however, that the number of trucks used is within a factor of 2 of the optimal solution.

Problem 2

Consider the following heuristic to solve the vertex-cover problem. Repeatedly select a vertex of highest degree, and remove all of its incident edges. Give an example to show that the heuristic does not have an approximation ratio of 2. Hint: try a bipartite graph with vertices of uniform degree on one side and with vertices of varying degree on the other side.

Problem 3

Consider the following simple greedy algorithm for the knapsack problem: sort the items by decreasing ratio of profit to size and pick objects in that order. Show that the algorithm can be arbitrarily bad!

Problem 4

Given a graph G with edge weights and an integer k, suppose we wish to partition the vertices of G into k subsets $S_1, S_2, \ldots, S_k$ so that the sum of the weights of the edges that cross the partition (i.e., have endpoints in different subsets) is as large as possible.

1. Describe an efficient $(1 - 1/k)$-approximation algorithm for this problem.

2. Now suppose we wish to *minimize* the sum of the weights of edges that do *not* cross the partition. What approximation ratio does your algorithm from part (a) achieve for the new problem? Justify your answer.