Cmpt-405/705

February 5, 2016

Assignment 4

Problem 1

Show that a maximum flow in a network $G = (V, E)$ can always be found by a sequence of at most $|E|$ augmenting paths.

Hint: Determine the paths after finding the flows!

Problem 2

Assume we are given a connected graph $G = (V, E)$ where all edges have different weights. Show that the following is true: there exists only one minimum spanning tree.

Problem 3

Prove the *Integrality theorem:* if the capacity function c takes only integral values, then the maximum flow f produced by the Ford-Fulkerson method has the property that $|f|$ is integer-valued.
Moreover, for all vertices u and v, the value of $f(u, v)$ is an integer.

Problem 4

We call a flow f *even* if $f(u, v)$ is even for all edges (u, v). A flow is called *odd* if, surprise surprise, $f(u, v)$ is odd for all edges (u, v). Prove or disprove the following two statements.

1. If all capacities $c(u, v)$ are even, then the value of the maximum flow is even.
2. If all capacities $c(u, v)$ are odd, then the value of the maximum flow is odd.