Cmpt-405/705

January 27, 2016

Assignment 3

Problem 1

Assume we are given a connected graph $G = (V, E)$ where all edges have different weights. Show that the following is true: there exists only one minimum spanning tree.

Problem 2

Sometimes when calculating a spanning tree we are not interested in minimizing the total weight, but rather just guaranteeing that no single edge in the spanning tree has too large of a weight. A minimum bottleneck spanning tree T of a weighted, undirected graph G is a spanning tree where the maximum weight among all of the edges in T is minimized, as oppose to a minimum spanning tree where the sum of the weights of all of the edges in the tree is minimized.

- Prove or provide a counter-example: any minimum spanning tree of a graph G is also a minimum bottleneck spanning tree of G.
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Problem 3

Assume we have already found a minimum spanning tree T for a weighted, undirected graph $G = (V, E)$. We would like to be able to efficiently update T should G be altered slightly.

- An edge e_1 is removed from G to produce a new graph $G_1 = (V, E \setminus e_1)$, such that G_1 is still connected. Give an algorithm that uses T to find a minimum spanning tree for G_1 in $O(|E|)$ time.
- An edge e_2 is added to G to produce a new graph G_2. Give an algorithm that uses T to find a minimum spanning tree for G_2 in $O(|V|)$ time.

Problem 4

Consider an undirected graph $G = (V, E)$ with distinct nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of G, and that you have also computed shortest paths to all nodes from a particular node $s?V$. Now suppose each edge weight is increased by 1: the new weights are $w_e? = w_e + 1$.

- Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.
- Do the shortest paths change? Give an example where they change or prove they cannot change