Homework 6

Released on November 21, 2016

---

1. Another way to perform topological sorting on a directed acyclic graph $G = (V, E)$ is to repeatedly find a vertex of in-degree 0, output it, and remove it and all of its outgoing edges from the graph. Explain how to implement this idea so that it runs in $O(m + n)$.

2. Given a strongly connected, directed graph $G = (V, E)$, an *Euler tour* of G is a cycle that traverses each edge of G exactly once, although it may visit a vertex more than once.

(i) Show that G has an Euler tour if and only if in-degree(v) = out-degree(v) for each vertex $v \in V$.

(ii) Describe an $O(m)$ algorithm to find an Euler tour of G if one exists.

3. Show that Generic-MST (in Slide 13, week 11-2) can color all edges.

4. Let $G = (V, E)$ be a graph with edge weights $c : E \to \mathbb{R}$ and let T^* be a corresponding min cost spanning tree. For fixed $e \in E$, let $[\underline{c}, \bar{c}]$ denote the largest interval such that T^* remains optimal if c_e is changed to any other value $c \in [\underline{c}, \bar{c}]$. Describe an algorithm for computing $[\underline{c}, \bar{c}]$ (do not write pseudocode).

Hint: Distinguish between $e \in T^*$ and $e \in E \backslash T^*$.

5. Show that MST problem can be formulated as a maximum weight forest problem, *i.e.*, given graph $G = (V, E)$ and "nonnegative" edge weights w_e, find a forest F of G such that $w(F) := \sum_{e \in F} w_e$ is maximized. (You may assume that G is a connected graph.)