Homework 4

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1. Show that RADIX-SORT yields a correctly sorted sequence.

2. In the algorithm SELECT, the input elements are divided into groups of 5.

(1) Will the algorithm work in linear time if they are divided into groups of 7?

(2) Argue that SELECT does not run in linear time if groups of 3 are used.

3. Describe an $O(n)$-time algorithm that, given a set S of n distinct numbers and a positive integer $k \leq n$, determines the k numbers in S that are closest to the median of S (including the median itself).

4. Let $X[1..n]$ and $Y[1..n]$ be two arrays, each containing n numbers already in sorted order. Give an $O(\log n)$-time algorithm to find the median of all $2n$ elements in arrays X and Y.

5. *Fibonacci numbers* are defined as below: $F_0 = 0, F_1 = 1$ and $F_i = F_{i-1} + F_{i-2}$ for $i \geq 2$. Give an $O(n)$ time algorithm (pseudocode) to compute F_n (using dynamic programming and implement in two ways: top-down and bottom up respectively). Draw the subproblem graph. How many vertices and edges are in the graph?

6. Consider a modification of the rod-cutting problem in which, in addition to a price p_i, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts. Give a dynamic programming algorithm to solve the modified problem. You only need to show optimal substructure and the recursive formula (including boundary conditions).

7. Consider n items $1, 2, \ldots, n$: Each item has a weight w_i and a profit p_i for $i = 1, \ldots, n$. A truck of capacity W carries these items to market and earn their total profit. We require that the total weight of items on the truck cannot exceed W. Assume $w_i \leq W$ for $i = 1, \ldots, n$ The goal is to find a set of items that can be carried by the truck with maximum total profit. Give a dynamic programming algorithm to solve the problem. You only need to show optimal substructure and the recursive formula (including boundary conditions).
Hint: Define item set $I_n = \{1, 2, \ldots, n\}$ and consider an optimal solution for I_n. Distinguish two cases: $n \in OPT$ or $n \notin OPT$.