Homework 1

Released on September 13, 2016

---

1. Prove the statements or answer the questions.

   (i) Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$? Prove your answers.

   (ii) prove that $2^{10} \cdot n \log n = o(n^2)$.

   (iii) prove that $o(g(n)) \cap \omega(g(n)) = \emptyset$.

2. Let L_1 and L_2 be singly linked lists.

   (i) present an algorithm (with pseudocodes) that combines the two linked lists into one and analyze the running time of your algorithm.

   (ii) Explain how to modify the data structure such that combing L_1 and L_2 can be done in $O(1)$.

3. Let $f(n) = n!$. Write a (non-recursive) pseudocode which outputs

$$f(n), f(n-1), \ldots, f(1)$$

in $O(n)$ time.
Hint: you may use data structure STACK.

4. Consider the hash function: $h(k) = k \mod m$, where $m = 2^p - 1$ and k is a character string interpreted in radix 2^p (for some constant p). Show that if we can derive string x from string y by permuting its characters, then x and y hash to the same value.

5. Consider hashing with chaining and assume simple uniform hashing. Let n_j be the length of linked list $T[j]$. Prove that $\mathbb{E}[n_j] = \alpha$, where $\alpha = \frac{n}{m}$ is the load factor.
Hint: A standard approach works as follows. First define the related random variables, with the help of the indicator function, cf. lecture w2-1, slide 6. Second, represent n_j as a function of the random variables. Finally, compute the expected values.